

Demand Heterogeneity, Inframarginal Multihoming, and Platform Market Stability: Mobile Apps *

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We modify the standard model of platform indirect network effects to accommodate superstar applications, such as those found in consumer oriented entertainment platforms. We apply our model empirically to the newest and largest computing platform, mobile apps. Our estimates show that the distribution of application attractiveness – the element missing in the classical theory – is central to understanding platform market structure in this industry. Our estimates provide an explanation for the partially fragmented structure of the industry in the U.S., an approximate tie between Android Phones and iPhones in the U.S., with tipping out by smaller platforms. Our estimates also provide an explanation of the significantly less fragmented structure of the mobile platform market in some other rich countries. The intuition for stability of fragmented equilibrium is simple. If an application is very attractive to the users of all platforms, it will find it profitable supply to all platforms, i.e. multihome. The supply decision will be inframarginal if the application is adequately attractive and there are sufficient users on the platform. If inframarginal multihoming is sufficiently important as a supply behavior, stability of fragmented equilibrium arises. We derive a quantitative stability condition and use it to explain not only the persistent near tie between the two large platforms in the U.S. but also to explain tipping out by smaller platforms in the U.S. and by all but one platform in smaller rich countries. Our estimates show that, for the U.S., those apps which satisfy the bulk of user demand are inframarginal multihomers on iOS and Android (but not on smaller platforms). Even if many users were to switch from iOS to Android or vice versa, these developers would still supply both platforms – explaining stability.

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In platform industries, equilibrium in application markets is linked to equilibrium in the platform market. The customers attracted to a platform constitute the potential market for applications supplied on that platform. In turn, the set of applications available on a platform are partly responsible for making the platform attractive to customers. A large theoretical literature examines the conditions under which the resulting feedback loop leads the platform market to “tip” to a dominant platform. (Farrell & Klemperer, 2007). With strong enough positive feedback, an approximate tie between two platforms is an unstable equilibrium: small departures from it by either users or developers tend to lead to a tip.

We provide a new model of tipping and stability in platform markets based on the industrial organization of applications markets. In our model, applications can be heterogeneous in their attractiveness to consumers. Stability can arise even when just a subset of applications are highly attractive to all consumers, whichever platform the consumers may prefer. Under those assumptions, if each platform has enough consumers, the *attractive apps will multihome*. Adequately attractive apps will be profitable over a wide range of market sizes, so their decision to multihome will be *inframarginal*. Tipping is blocked because a change in the number of users leads to little change in application supply (it is inframarginal) especially among the applications most attractive to consumers; we derive a sufficient condition for stability and provide an analysis of market circumstances in which it will likely hold.

We apply our model to the newest, and already largest, platform market: mobile apps running on smartphones in the US. The model provides an empirical explanation of non-tipping between the two leading US market platforms. It is also consistent with the tipping-out of smaller US platforms and with the tendency of many national markets to have tipped to a dominant platform. This paper is the first to study the new mobile platform market empirically, and the first to examine the importance of applications attractive to many users, a key feature of mass-market platforms.

A new successful platform creates a new applications industry. The economics of supply and demand in those applications markets will, as they do in all industries, lead to an industry structure, perhaps atomistic, perhaps highly concentrated. The mobile revolution opened up a new frontier for computing, with the consumer as the primary user. The resulting industry structure for applications takes on some of the aspects of other mass-market entertainment and communications industries, like music or social networks. We shall see that app demand is highly concentrated, with a modest number of products attracting dramatically more customers than the others. Second, there is a strong tendency of users of one platform to demand the same apps as users of the other platform, at least among the highly-demanded apps.

We estimate a new model of user demand for apps and developer choice of platforms. We combine hand-constructed data on developers’ platform choices and app and developer characteristics with commercial data on app usage. This new dataset plus our model lets us estimate the developers’ expected demand from

entering either or both platforms. It also lets us infer user demand for apps on both platforms.

The inframarginal multihoming result is the key to explaining the stability of the fragmented platform market equilibrium. Stability can arise either because users do not respond strongly to app availability in choosing a platform, or because developers do not respond strongly to the number of users on a platform when making supply decisions. Our explanation for stability, which does not appear to be in the theoretical literature on tipping, is of the second form. We show that heterogeneity in applications demand has important implications for determining which developers will be inframarginal with respect to the number of users on a platform when deciding whether to supply to one or more platforms. In our setting, the structure of demand and the industrial organization of the developer sector leads to an equilibrium in which most developers, including those most valuable to consumers, are inframarginal to both platforms. Stability follows.

We contribute a new analysis of the conditions for tipping vs. stable platform market fragmentation. Inframarginal decisions tend to lead to stability while marginal decisions, if aligned with a feedback loop, tend to tipping. This condition shows the importance of application demand heterogeneity. We believe we are the first to empirically document how the incentives to multihome for even a few very popular apps in a fragmented market can sustain fragmentation. These features of the economics of *complementors* appear to be critical to platform market competition.

Another surprising finding is a difference between established firms and entrepreneurial app developers. Despite expectations that the massive entry due to platform sponsorship would lead to disruptive entrepreneurial innovation, we find that the most demanded apps tend to be from established firms. Our discussions with industry participants suggest that the first-order challenge for apps in the industry today is how to get discovered out of the clutter of apps available. Entrepreneurial firms face a high cost of marketing to one, let alone two, platforms. Established firms are able to avoid this cost thanks to their existing customer relationships. In this setting, the potential for disruptive entrepreneurial innovation is diminished. Again, the economics of the complementors influence the platform market competition: the most popular apps tend to be from established firms with no incentive to promote platform tipping.

The paper is structured as follows: Section I reviews the relevant literature. Section II describes the industry setting of mobile apps. Section III describes our data. Section IV describes our model of developer choice over platforms and multihoming. Section V describes how we implement the economic model to analyze our data. We discuss our findings in Section VI and identification in Section VII. The last section concludes.

I LITERATURE REVIEW

Our analysis contributes to the longstanding literature on platform economics and to the nascent literature on the mobile industry.

I.A Classical Indirect Network Effects Theories

A large theoretical literature, beginning with Katz and Shapiro (1985) and summarized in Farrell-Klemperer (2007), lays out the basic framework of indirect network effects we use. A central idea of the literature is the positive feedback loop. If users choose the platform with the most applications, while developers choose the platform with the most users, equilibrium can follow the positive feedback loop and “tip” to a single platform.

Positive feedback is not sufficient for tipping. Whether a platform market is “tippy” depends the strength of the feedback loop among users and developers. A market will tend to be tippy if users care strongly about the availability of applications and developers care strongly about access to users. A market will be less tippy if users or developers have stronger and more differentiated tastes for the platform itself. If users mostly choose the platform that has all the cool apps (and agree on what that is) the market is tippy. If users mostly choose the platform that they think itself is cool (and disagree about whether that is iPhone or Samsung) then it will be less tippy. Fragmented equilibria arise when platform product differentiation is a stronger force, tipping arises when network effects are a stronger force.

We will follow this approach, relaxing one of the key assumptions of the models, i.e. that users value the *number* of applications available on a platform and developers value the number of users who are their potential customers on a platform. Rysman (2004) calls this the “representative developer” (“representative advertiser” in his application) model. Our contribution in Section IV is to add developer heterogeneity in the sense that some applications are much more valued by users than other applications. For a mass consumer market like mobile apps, as for any other platform context with strong elements of consumer entertainment and consumer-to-consumer communication, the possible presence of superstar applications is an important possibility to consider. Like the classical literature, we will examine a stability condition to understand the economics of a fragmented platform market equilibrium vs. one that tips to a dominant platform.

The literature has also noted that competition among the platform providers is important.¹ Whether platform providers differentiate or match one another is a determinant of the strength of the tipping forces, of course. Platform providers can differentiate on the developer side, by offering different technical features,

1. Brown & Morgan (2009) and Cantillon & Yin (2014) look at platform product differentiation in a variety of dimensions as a determinant of complementor platform choice, drawing inferences about platform market structure.

on the user side, by offering different qualities such as ease of use, and on both at once, by targeting a subset of users and the applications they value. Similarly, whether platform providers use aggressive pricing strategies can determine whether a market tips. While our empirical analysis will take the actual platform provider choices into account, strategic platform provider behavior remains largely in the background in our analysis.²

I.B Indirect Network Effects Empirics

A number of papers have empirically investigated some aspect of the positive feedback loop. Greenstein (1993) measured “installed base effects.” Gandal, Kende and Rob’s (2000) estimates distinguish between two elasticities of demand for a platform good: the own-price elasticity, related to platform product differentiation and the applications-variety elasticity, related to the positive feedback loop. Augereau et al.(2006) examine strategies of coordinating on a particular standard (i.e. a platform) vs. differentiating. Rysman (2004) is an important paper that goes beyond this to investigate platform market equilibrium. Rysman estimates behavioral equations for all three classes of actors, and thus can investigate the equilibrium tradeoff between platform competition and the exploitation of indirect network effects. Rysman’s strategy relies on seeing the equilibrium of the same platform market in a large number of localities; almost all applications, including ours, will not observe the same market both having tipped and having not tipped. This equilibrium restriction leads us to our bounds strategy.

All these papers use a “representative developer” (or representative advertiser to Rysman) paradigm, treating developers as if they were all equally attractive to consumers.³

I.C Multihoming

There is a substantial literature on multihoming in platform industries. Much of this literature has been concerned the effect of multihoming on platform competition and market power (Rochet-Tirole (2003). Rysman (2007) reviews this literature and provides an empirical analysis of multihoming and single homing in credit cards, providing suggestive evidence of positive feedback helping larger platforms (e.g. Visa) and not smaller ones (e.g. Amex) without undertaking an equilibrium analysis.

A number of papers discuss multihoming. The theoretical literature is well summarized in Rochet and Tirole (2003). Empirically, the credit-cards analysis of Rysman (2007) takes up multihoming on both the merchant (accepting credit payments) and consumer sides. The video-games analysis of Corts and Lederman

2. The user-side pure platform product differentiation forms an important part of our stability bound; platform product differentiation toward developers is both difficult to estimate and largely irrelevant when the most attractive developers multi-home.

3. Gandal, Kende and Rob measure “variety” with the number of titles, following the classical theory.

(2009) studies multihoming by game developers. Both of these papers report that the costs of multihoming are very low for at least one side of the market, rendering indirect network effects significantly less important. We, in contrast, study an industry in which essentially all users single home but the decision to multihome divides developers, so that markets can be either tippy or not.

I.D Content

Rosse(1970) and Berry-Waldfoegel (1999) examine “content” platforms with consumers as the customers. An important idea from the study of content industries is the role of superstar products, i.e., of the heterogeneity in attractiveness of content to consumers – the phenomenon of “bestsellers” in Sorenson (2007), for example. While Berry-Waldfoegel examine a positive feedback phenomenon somewhat like tipping, to our knowledge no studies of content industries have yet taken up the relationship between superstar/bestseller market structure in the applications markets and platform market structure.

I.E Two Sided Markets

A related literature, typically referring to “two-sided markets”, emphasizes the role of platforms as market intermediaries.⁴ Another literature with similar concerns examines the contractual relationships between platform providers and complementors.⁵ These contracts can help to overcome coordination problems and focus developers and users on a particular platform, a new platform, etc. Sometimes, these contracts are exclusive, i.e. limit supply of an application to one platform. In some circumstances, the structure of these contracts is very important to platform competition, and both an empirical and theoretical literature has engaged with them. We will touch on such contracts only peripherally; they are salient to some aspects of our industry but not to our central question of fragmentation.

II INDUSTRY SETTING: MOBILE APPS

The platform industrial organization underlies the latest large-scale information technology industry, mobile apps. The invention of mobile app platforms has permitted developers to offer a system solution to their customers – re-using, not re-inventing, the technology of mobile phones, mobile transmission, wi-fi,

4. This label for the distinction between the two literatures comes from Rysman(2007). We care more about the distinction than the label.

5. The largest body of empirical papers examines the economic or contractual relationships between complementors and platform sponsors, one of the core concerns of the theoretical literature (Caillaud & Jullien, 2001, 2003; Rochet & Tirole, 2003; Armstrong, 2006). Some of these papers take up platform policies, such as openness, exclusivity, and vertical integration, in the context of platform competition (Boudreau & Hagiu, 2009; Corts & Lederman, 2009; Boudreau, 2010; Dube *et al.*, 2010; Lee, 2013; Halaburda & Yehezkel, 2013; Jeitschko & Tremblay, 2015).

cloud technologies, and many other components.⁶

Effective use of the platform organization has permitted this industry to grow faster than earlier IT markets, and today there are more smartphones running apps than there are PCs running applications. In this section, we review the basic background of the industry, providing an explanation of our main modeling choices and demarking what we study from what we do not study.

There have been smart cellphones for decades, but the current mass market consumer smartphone industry was founded with the 2007 introduction of the iPhone. Before that, RIM dominated a much smaller market with its Blackberry devices for business communicators and with programming frameworks used extensively by employers and by a group of business-oriented app developers. The iPhone came with a more consumer-friendly design, and from the beginning was connected to the existing entertainment products (notably music) in the iTunes store. The iTunes store also provided a distribution venue for new consumer apps.

The combination of a mass market of consumers, a distribution channel, and the need to write only software, with many hardware (e.g. the iPhone handset itself) and communications (e.g. the mobile cell networks) assets already in place, drew a wide variety of app developers. Today, there is a wide variety of apps and app developers, most seeking to serve a mass-market audience. The largest market category is games, which are primarily supplied by entrepreneurial firms founded in the mobile era (e.g., Rovio Entertainment, maker of Angry Birds). (Yin *et al.*, 2014). Entrepreneurial firms have also built complex mobile systems, such as the Uber ride sharing service. Most early developers were entrepreneurial and, to the extent they had market success, leaned toward consumer entertainment and information services. A number of other apps were officially sponsored as part of the smartphone platform. These were largely extensions of existing online products and services, such as web browsing, email, and maps into the mobile world.

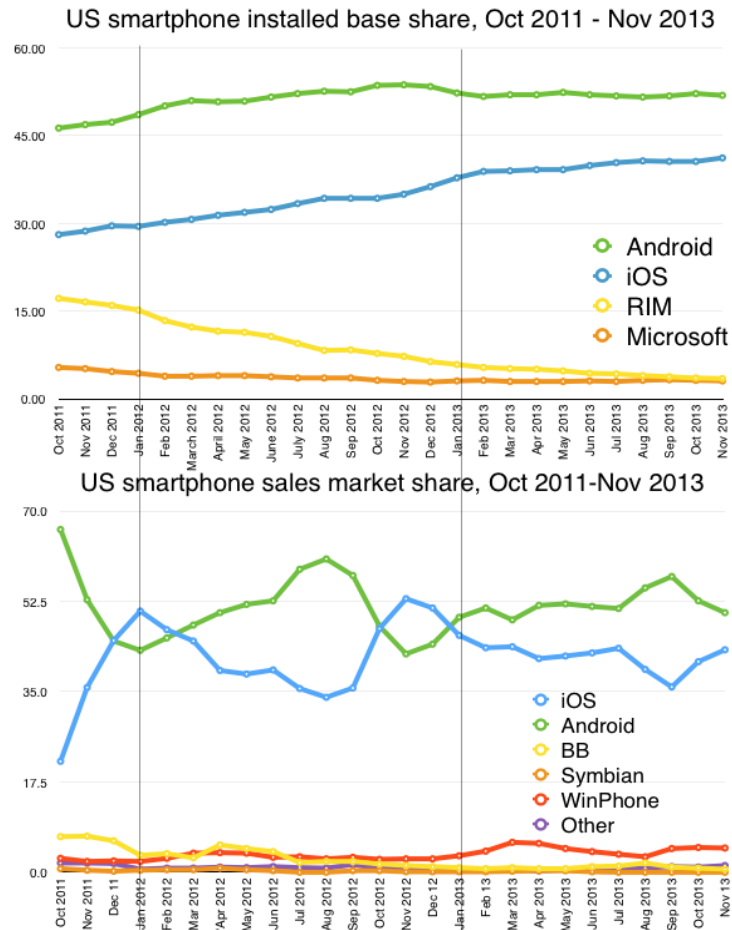
There was quickly a powerful positive feedback loop around the iPhone. Within a year, check the installed base of iPhones exceeded that of Blackberries. Within a few years, the installed base of Blackberries began to decline.

A second new consumer-oriented smartphone platform, Google’s Android, was introduced 16 months after the iPhone. Google adopted an open systems strategy for its platform, and that offered contrasts to Apple’s approach along several dimensions. These contrasts mattered for the period during which Google caught up to Apple – one assumes that was their point. However, we emphasize the ones which still matter for today’s platform market.

Apple is vertically integrated into *handsets*; users get an iOS smartphone from Apple. Android smart-

6. For a more in-depth review, see Bresnahan, Davis, & Yin (2015).

FIGURE 1: FRAGMENTATION OF USERS IN US



Source: http://www.tech-thoughts.net/2012/07/global-smartphone-market-share-trends.html#.UtdYR_RDtnJ

phones come from a wide variety of sellers. This openness enabled product variety; the least expensive Android phones, for example, have been and are significantly cheaper than any new iPhone. It also enabled experimentation by many different sellers. Experimentation changed the name of the leading Android manufacturer and let the best Android phones catch up to iPhones. This product variety and rapid technical progress (at the top) drew users to Android rapidly. Today, we are used to the idea that Samsung phones at the top of the product line rival iPhones in feature and quality, and that very cheap Android phones are available worldwide.

Apple imposed a series of strong contractual restrictions on developers. Apple mandates distribution only through the iTunes store. This permits Apple to run an approval process for apps it finds suitable for its customers. Apple restricted the programming languages developers could use to write apps. In contrast, entrant Google permitted broad freedom in developing and distributing apps, to the point of committing to permit distribution of Android apps outside the Google Play Store, that is, outside of Google’s control. These pro-developer strategies led to a rapid expansion of Android apps, catching up to iOS apps after a period of time.

Many platform differences, adopted in a period of initial dominance and catch-up, matter today. Today, developers tell us, the handset cost differences mean that iPhone users are on average richer than Android users. Many developer monetization strategies, such as selling advertisements, charging users for an upgraded app (“freemium”), charging users for smaller app improvements (e.g. a bigger sword in a warlike game) through in-app payments (“IAP”) or selling users something in an app that is basically a store are forecast by developers to be more lucrative on iOS. Our model will need to permit this.

Effective competition from Android led Apple to relax some of its developer restrictions. Similarly, some of Android’s early shortcomings as a development platform for commercial apps, such as weak payment systems, have been overcome. But the open-systems Android platform has the weakness of one of its strengths – the wide variety of Android phone screen shapes and sizes means that developers bear additional user-interface development costs on that platform. User-interface development costs are typically a large fraction of the costs of writing a mass-market app. Today, there is a lively debate among developers about the relative costs of writing an effective app for each major platform. Our model will need to permit this, too.

Some rich countries have market structures like the US, fragmented between the two leading platforms. Most other countries, however, have different platform market structures, with less fragmentation and significantly higher Android market shares. This includes all the poor countries, and a few rich ones such as France and Germany. It is very difficult to study a market that has tipped to a dominant platform,

as almost all agents are far from any choice margin.⁷ There appears to be little commercial interest in users of the minority platform when it has tipped out: commercial databases covering the minority platforms in rich countries, including Apple in many places and Windows Phone and Blackberry everywhere, are not available.

We study smartphones, not mobile tablets. Platform market structure in tablets is significantly more complex and difficult to study, so we leave it for a better day and better data availability. Two difficulties come with the definition of “tablet.” Google’s open-systems strategy permits Amazon to run a parallel distribution system, so that Amazon Kindles both do and do not share apps and other complements with “regular” Android tablets. Most US Android tablets are Kindles. Other tablets run Windows 8, like the Microsoft Surface and some machines from other OEMs. These are mixtures of laptop PCs and tablets, another complication. Comparable app-demand data for these tablets are in any case limited.

There is extensive public discussion of mobile apps, but as yet relatively much less statistical work.⁸ Journalists, bloggers and other industry observers accurately reported that, after a period of experimentation with paid apps, most apps today are free at time of download (though they may create value for sellers through freemium, IAP, advertising, or through selling other services. (For systematic measures of monetization and other aspects of app developer strategy see Bresnahan, Davis, & Yin (2015)). So our empirical model will ignore app pricing.

The public discussion has heavily emphasized a few extremely successful entrepreneurial app developers, including Rovio (the Angry Birds series of games), Uber (ride sharing), and WhatsApp (evasion of mobile phone carrier messaging charges). We are concerned that this focus may be only partly right, and that our empirical model of app demand should not be too much guided by it. First, the existence of these superstar apps raises the question of the shape of the distribution of app attractiveness to users. Our empirical model will need to fit that distribution.

Here, we are also concerned that the public debate’s focus on entrepreneurial apps may miss an important category. In our earlier work, we documented the importance apps from established, as opposed to entrepreneurial, firms. These apps support consumer product and services companies in their main lines of business, notably airlines, banks, and retailers. It will be important for our empirical model to permit entrepreneurial and established apps to play different roles, not to assume that one is more important than the other.

All these features of app supply and demand are related to platform market equilibrium, of course. We emphasize the differences across apps because of the implications of our platform equilibrium model with

7. We will return to the conjecture that the market may have tipped against iOS in these countries in our conclusion; our estimates, though based in the U.S., will have something to say about this question.

8. *Add cites.*

heterogeneous apps. What that model directs us to measure is a slightly complicated object, but an intuitive one, the supply behavior of the developers of apps that users want. Those are the apps that draw users to a platform.

Consumers tend to single-home, choosing only one smartphone platform. Apps cannot be used across platforms (an iOS app can be used by iPhone users but not by Android, Blackberry or Windows Phone users); however, developers can incur a cost to develop their app for each of the multiple platforms. Our discussion with developers suggests that porting to another platform is not technically difficult. Instead, the largest costs of multi-platform supply are the marketing costs to reach a new population of users on the other platforms. Before even being considered for download by consumers, an app needs to attract the potential user's attention out of over 1 million apps on either Google Play store or iTunes App store. Launch campaigns average approximately \$0.5 million. Entrepreneurial app developers tell us that to gain attention they buy ads displayed in other app developers' apps and pay for "incentivized downloads" in an effort to gain visibility in a mass market.

Other types of developers tell us they face very different marketing costs. Developers with lines of business outside of mobile (e.g., Facebook, Starbucks, United Airlines) typically integrate their app into existing lines of business (e.g. an airline app displays boarding passes). These developers typically offer their app to existing customers. Our empirical model will need to let marketing costs vary by developer type.

A strand of the platform economics literature emphasizes contracts between developers and platform providers for exclusivity, for coordination, etc. These matters are not particularly relevant to our empirical enquiry. Almost all apps are supplied by third party developers, and relationships between developers and platform providers are arms-length and market-like. A few apps are provided by platform providers in-house (Google Maps, Talk, Calendar, Gmail, Apple Weather, Contacts, ...). With very modest exceptions, the two largest platforms do not have exclusive contracts with outside developers.⁹ Complex contracts between developers and these platform providers are absent.¹⁰

The Blackberry platform was dominant in smartphones in an earlier, business-user oriented era.¹¹ Blackberry had almost all of the users and apps of the earlier era, but the size of that installed base was overwhelmed by the mass-market consumers served by the Apple iPhone and its entertainment and media apps. Blackberry switched to a more consumer-friendly strategy after a period of time, but found itself in a downward tip with too few users to attract apps and too few consumer-oriented apps to attract users. Microsoft

9. A few predictably popular games have been exclusive to a platform for a short period of time at initial launch but not long-term.

10. Apple has an app approval process that it uses to influence some developers' app content. The main directions of influence are consumer-protection and preventing duplication of what Apple deems to be its core services. Comparatively open-systems Google has only a light review process.

11. See Bresnahan & Greenstein (2014) for more complete analysis of this incident and for cites to industry sources.

was also a late entrant into consumer-oriented mobile platforms, and found itself with tipping forces pushing down its demand. Microsoft has adopted a number of strategies to deal with shortage of mobile apps. These include paying small bounties to each developer who submits an app, and very large bounties, reportedly up to \$100k, to selected developers.¹² This expensive program has been sufficient to keep Windows phone from completely dying off, although its shipments are about 1/20th of those of Android. While these two smaller platforms will not play a role in our econometric analysis, since industry sources do not find it worthwhile to collect data on them, nonetheless their history shows the importance of apps to platform suppliers and the tippiness of the platform market.

III DATA

We have gathered data on apps which were written for either iOS (iPhone, denoted by i), or Android phones (denoted by d), or both (denoted by b). We begin with a commercial dataset, the January 2013 Mobile Metrix dataset from comScore.¹³ This dataset reports a number of marketing variables at the app-platform level. Throughout, we will use the $*$ notation to denote comScore variables. Our measure of demand from comScore is r^* , the fraction of users who run the app, called “reach” in the industry. r_{pa}^* is the reach of app a on platform p . Our measure of supply from comScore is S^* , a dummy variable for the observed presence of the app. “Observed” is important here, since S_{pa}^* , the observed presence of app a on platform p , can be zero even if the app has been supplied for the platform. comScore bases these data on two samples of users, one with Android phones and the other with iPhones. Each sample has 5,000 US adult users. Apps are only reported – and therefore only observed in our data – if they have more than 5 unique users on the platform in the sample.¹⁴ This means that r_p^* is truncated from below at .001. It also means that all apps in our sample have comScore data on at least one platform, i.e., have $S_{pa}^* = 1$ for either $p = d$ or $p = i$. Since we are interested in supply by “independent software vendors,” we restrict our sample by excluding apps produced by Apple, Google, carriers, and OEMs. This yields our analysis sample of 1,044 apps.

Definitions and descriptive statistics for our data from comScore are in Table 1. Figure 2 presents a graphical display of the joint distribution of r_{da}^* and r_{ia}^* for apps where both are observed (i.e., $S_{ba}^* = 1$). It also identifies the top apps by name. While the figure is simply raw data, subject to selection bias, it

12. See Bass & Satariano (2010). Other Microsoft strategies have included buying Nokia and creating programming frameworks that permit apps to be shared between Windows PCs and mobile devices. The ease-of-porting strategy has worked significantly better for Microsoft tablets than for smartphones.

13. This dataset is available for subscription at academic rates. We will provide our programs for processing it to anyone seeking to replicate this paper, but you will need to buy your own copy of the underlying data.

14. comScore also reports a few apps with less than this level of usage if a client has requested tracking. We drop these apps.

TABLE 1: DESCRIPTIVE STATISTICS FROM COMSCORE DATA

Variable	Mean	Std Dev	min	max	N
S_i^*	0.574	0.495	0	1	1,044
S_d^*	0.657	0.475	0	1	1,044
S_b^*	0.231	0.422	0	1	1,044
r_i^*	0.021	0.060	0.001	0.775	599
r_d^*	0.018	0.050	0.001	0.735	686
$corr(r_i^*, r_d^* S_b^* = 1) :$			0.691		241

Definitions: S_i^* , S_d^* , S_b^* are indicator variables for whether the app was observed on iOS, Android, or both platforms (these are not mutually exclusive categories). r_i^* and r_d^* report the reach of apps on each platform, when observed. The last line reports the correlation between reaches when the app is observed on both platforms.

shows that demand for a particular app on one platform is correlated with the demand on another. Only a few highly-demanded apps are much more popular on one platform than the other; these outliers appear to reflect special supply circumstances.¹⁵

We also manually collected data from public sources such as the online app stores, developers' websites, Crunchbase, and the app itself. There are no systematic data available on developer and app characteristics or on developer supply decisions in this industry, so these data require hand work.

Table 2 reports data from this hand effort. We call S_{pa} the true supply of the app on a platform, to contrast it with S_{pa}^* , the supply of an app on a platform as reported by comScore, which is censored. The table also reports the incidence of true multihoming. Comparing Table 2 and 1, we see that 65% of apps multihome, while only one-third of those actually reach enough users on both platforms for us to observe them in comScore (the mean of $S_b^* = 0.231$).

15. The most popular outliers (away from the 45° line) are from Yahoo. Each is competing with platform-sponsored apps which are effective on only one platform. Yahoo Messenger competes with the Google Talk and with no effective Apple-sponsored product. On the iPhone, Yahoo Weather has been repackaged as the iPhone's native weather app.

FIGURE 2: JOINT DISTRIBUTION OF OBSERVED REACH r^* FROM COMSCORE JANUARY 2013 ON ANDROID AND IOS FOR $S_b^* = 1$ (LOGSCALE)

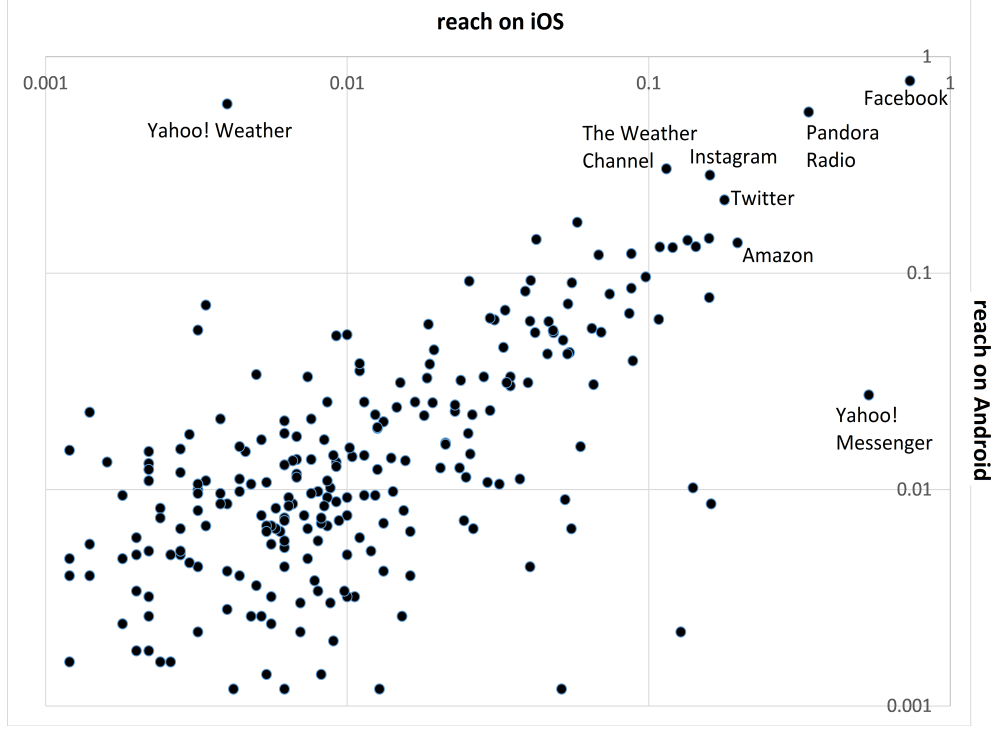


TABLE 2: DESCRIPTIVE STATISTICS ON APP DATA (N=1,044)

	Variable	Mean
Y	S_i	0.765
	S_d	0.820
	S_b	0.647
X App Characteristics	Game	0.313
X Firm Characteristics	Mobile Only Firm	0.420
	Online Firm	0.290
	Offline Firm	0.290

Note: All variables are indicators. Sample is all third-party apps observed in comScore on at least one platform.
 $S_p = 1$ if $p = \{d, i\}$ if the developer's website states the app is available on platform p and provides an appstore link.
 $S_b = 1$ if $S_i = 1$ and $S_d = 1$.

Game=1 if the app is in the game category.

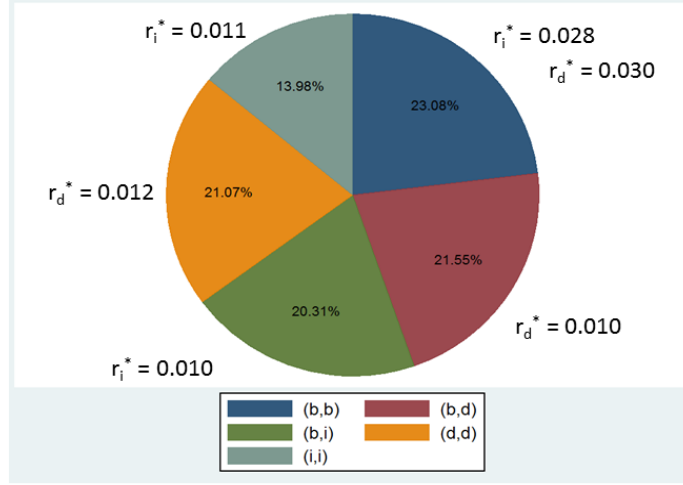
Mobile Only=1 if the developer's only line of business at firm founding involves mobile apps. (Some mobile-era firms, such as Rovio, have added other lines of business after entry, but we classify them as "mobile only" by their status at entry.)

Online=1 if the developer has an online business along with a mobile app (e.g., Facebook) and was, at time of entry, an online-only firm.

Offline=1 if the developer had an offline business before having an online business or mobile app (e.g., Delta, Nike, other brick and mortar stores selling physical goods or service).

Mobile Only + Online + Offline = 1

FIGURE 3: PLATFORM CHOICES AND MEAN REACH
FREQUENCY OF (S_p, S_p^*) AND CELL MEAN OF S_p^*



We use characteristics associated with app a or its developer, X_a , as regressors. All our regressors are indicator variables; descriptive statistics are in Table 2. We use only one app variable, *Game*, which is the only app classification variable from either comScore or the app stores which is measured reliably. Based on the findings of Bresnahan, Davis, & Yin (2015), we base our three firm types on the technological era in which the firm entered: offline, online, and mobile.

Figure 3 shows the distribution of true and observed app supply (S, S^*) , which can take on five values. For example, the notation (b, d) means that the app was written for both platforms but crosses the comScore threshold only on the Android platform. The cells are approximately equally frequent, with only i, i , single-homing on iOS, departing. Adding up the sections in Figure 3 shows that supply decisions are largely symmetric. First, almost two thirds (0.647) of developers in our sample multihome. Second, among all developers in the sample, just over 80% write for Android and just under 80% write for iOS. The means of observed reach are very similar in four of the five cells, at around 0.01. The (b, b) cell, i.e. observed multihomers, has much larger reach at around 0.03 for each platform. Interpretation of this figure depends on unraveling selection; nonetheless, it is clear that one interesting tendency is for more popular apps to multihome.

IV ECONOMIC MODEL OF PLATFORM INDIRECT NETWORK EFFECTS

To examine the conditions for stability of a platform market equilibrium, we begin with the incentives of developers and users following the literature summarized in Farrell & Klemperer (2007).

IV.A Developers

The developer of an app chooses the platform(s) on which to publish. A particular app may be attractive to users on one or both platforms. If app a is published on platform p , it earns profits¹⁶

$$\pi_p = M_p \times U_p \times r_p - C_p, \text{ where}$$

M_p is the per-customer margin earned by app a on platform p ,

U_p is the number of users of platform p ,

r_{pa} is the reach of app a on platform p , and

C_p are the fixed costs of writing and marketing the app.

This is an entry model, where developers pay a platform-specific fixed cost to enter the market. Indirect network effects appear in the number of customers for the app on platform p , $U_p \times r_{pa}$. The overall attractiveness of the platform to consumers determines U_p , while the attractiveness of the particular app to users on platform p determines r_{pa} . Platform supply for an individual developer is:

$$(1) \quad S_{pa} = 1 \iff M_p \times U_p \times r_{pa} > C_p \iff r_{pa} > \hat{r}_p \equiv C_p/M_p/U_p$$

where $\hat{r}_p$ is the entry threshold. For indirect network effects analysis, we emphasize its dependence on U_p and write $\hat{r}_p(U_p)$. A developer that has $r_{pa} \gg \hat{r}_p$ is an *inframarginal supplier* to platform p . A developer that has $S_{da} \times S_{ia} = 1 \equiv S_{ba} = 1$ is a *multihomer*. A developer that has $r_{pa} \gg \hat{r}_p$ for both p is an *inframarginal multihomer*.

The aggregate supply behavior of all developers, can be derived from equation (1) and the distribution of developer types. We model developers as varying in their attractiveness to users (reach). The cumulative distribution function of $r_a \equiv (r_{da}, r_{ia})$ is $F(r_a)$. Dropping the a subscript for clarity, this has marginal densities $f(r_p)$. Equation (1) yields not only the supply behavior of each developer, but the aggregate supply of apps to each platform, $A_p(U_p) = \{a \mid r_{pa} > \hat{r}_p(U_p)\}$. Together with $F()$, equation (1) determines the mass of inframarginal suppliers to each platform, of multihomers, and of inframarginal multihomers.

16. Here we assume that all apps have the same margins and costs on a platform. The case in which these are allowed to vary across classes of apps has the same logic but is much more clumsy to write. In our empirical work, we permit costs and profits to vary across app classes and reflect this in our stability calculations. Our empirical model also distinguishes between a developer's signal of expected reach on a platform and realized reach, which is included in our empirical stability calculations below but suppressed here.

The distribution of developer types F determines a number of aggregate supply elasticities. The elasticity of total supply to the platform per user with respect to the threshold is

$$\eta_p^S \equiv -f_p(\hat{r}_p)\hat{r}_p \frac{\hat{r}_p}{\int_{\hat{r}_p}^1 r f_p(r) dr}$$

This reveals the key features of developer heterogeneity for the supply elasticities. Marginal developers supply $\hat{r}_p f_p(\hat{r}_p)$ apps per user on platform p . Developers who are inframarginal to platform p (who may also be inframarginal to both platforms (inframarginal multihomers)), supply $\int_{\hat{r}_p}^1 r f_p(r) dr$ apps per user on platform p . We estimate F and the related elasticities in the empirical section.

IV.B Users

Users pick a single platform based on its intrinsic value $\bar{v}_p$ and the value of apps available on it $v_p = v(A_p)$. By the intrinsic value of the platform we mean users' valuation of the quality and variety of available devices (phones), platform services, ease of use, the price of the platform good, and so on. In our platform-stability analysis, this is given and exogenous.¹⁷ Since users choose apps that run on their platform based on the same preferences they use to choose the platform itself, we model the app-valuation function v_p as increasing in the attractiveness to users of the apps available on a platform.

We model $v(A_p)$ as a *quantity index* of the value of the apps available to users on platform p .¹⁸ We assume

$$(2) \quad v_p = v(A_p) = \int_{a \in A_p} r_{pa} dF_p(r) = \int_{\hat{r}_p}^1 r f_p(r) dr,$$

This index reflects the idea that those apps which attract many users make a large contribution to the attractiveness of the platform to users. The densities $f_p()$ that enter here are the same as those we introduced in the developer supply section. The point here is the link between user demand and heterogeneity in developer supply. A developer whose application is very attractive to users on a platform will be both an inframarginal supplier and a large contributor to v_p . A developer whose application is very attractive to users of multiple platforms will be an inframarginal multihomer and a large contributor to each v_p .

The number of users on each platform – user demand – is a function of the intrinsic values of all platforms ($\bar{v} \equiv \{\bar{v}_p\}$) of the user value of the apps available on all platforms ($v \equiv \{v_p\}$) and of the total number of

17. Our model is a treatment of the stability of the equilibrium of the game among users and developers given platform providers' offerings.

18. Since most apps are free, a price-index approach to valuation is not feasible. If there were prices, one could use the methods of Small & Rosen (1981) to calculate an indirect utility function of the apps on a platform.

users U_M :

$$(3) \quad U_p = D_p(\bar{v}, v, U_M)$$

The demand elasticities with respect to app availability are $(\partial D_i / \partial v_j)(v_j / D_i) \equiv \eta_j^i$, where i and j are platforms (including the case $i = j$). We also assume $(\partial D_p / \partial U_M)(U_M / D_p) = 1$, i.e., if there is a larger population of users it has the same mix of tastes. We will also assume symmetry in demand slopes, i.e. $\partial(D_p) / \partial(v_{p'}) = -\partial(D_p) / \partial(v_p)$, as would follow from discrete-choice demand for platforms.

IV.C Platform Market Equilibrium and Stability

We now examine the conditions for stability of a symmetric equilibrium. A platform market equilibrium consists of U_p and $\hat{r}_p$ that solve Equations (1) and (3) for all p . Since users single-home, we write the equilibrium condition as a fixed point a mapping from U to itself.¹⁹ Stability can be determined by looking at this mapping.

We define the mapping $U = \chi(U)$ whose fixed points are equilibria using Equations (1), (2), and (3). For this purpose it is useful to define the variables U , v and $\hat{r}$ as the vectors whose typical elements are U_p , v_p and $\hat{r}_p$, respectively. We can rewrite demand for all platforms following Equation (3) as $U = D(\bar{v}, v, U_M)$.

We replace the endogenous variables v with their definitions from Equation (2), which determines v_p as a function of $\hat{r}_p$, writing this as $v(\hat{r})$. Thus we have $U = D(\bar{v}, (v(\hat{r}), U_M)$. Finally, we note that $\hat{r}_p(U_p)$ comes from the supply behavior of developers, (1), and make the vector definition $\hat{r}(U)$. Thus, equilibrium is a fixed point of $U = D(\bar{v}, (v(\hat{r}(U)), U_M)$, a function from U to itself.

If both demand and supply are symmetric, a symmetric equilibrium will exist.²⁰

Any equilibrium will be *locally stable* if, at that equilibrium, the function $U(U)$ just defined is not too steep, more precisely, if the real part of all eigenvalues of the function are less than 1 in absolute value at an equilibrium, the equilibrium is stable. The jacobian of $U = D(\bar{v}, (v(\hat{r}(U)), U_M)$ can be written in terms of the underlying functions, $J_D J_v J_{\hat{r}}$.

This is the same logic as a familiar local stability condition. The only difference from classical models is our treatment of heterogeneous apps in value to users (2) and in supply (1). If, for example, the greatest mass of attractive apps are inframarginal multihomers at a particular candidate equilibrium, $\partial v_p / \partial \hat{r}_p * \partial \hat{r}_p / \partial U_p$ and will be small, so $J_v J_{\hat{r}}$ will be a small diagonal matrix. Then equilibrium will be unstable only if J_D is large, i.e., if users respond strongly to changes in v .

19. This useful shortcut is common in the literature; see Farrell and Klemperer (2007).

20. "Demand is symmetric" means two things: (D1) Platform demand by users (U_p) is symmetric in v and $\bar{v}$ (D2) Users' app demand is symmetric across platforms. "Supply is symmetric" has a cost/margins element, (S1) $C_i / M_i = C_d / M_d$ and a potential product quality element, (S2) $f_d(r) = f_i(r)$ (equally popular apps potentially available to both platforms).

Our estimates permit us to estimate $J_v J_{\hat{r}}$ directly.²¹

We cannot estimate $D()$, so we use our estimates to provide a *stability bound*. Our strategy for this is to assume that $D()$ takes a logit form and bound its elasticities. In particular, we bound η_i^i , the elasticity of user demand for iOS with respect to changes in v_i , the attractiveness of the applications available on that platform. We find the smallest η_i^i that would let equilibrium be locally unstable. The shape of the distribution of apps' attractiveness, $f(r)$, enters the stability condition through η . If lowering the app developer entry threshold (decreasing $\hat{r}$) leads to a large increase in user value, η will be large. Alternatively, if η is small, it means that most of the user value comes from inframarginally supplied apps. At the historic equilibrium in the US, that turns out to be a very large elasticity, as we shall see below. The point is, in such a large market as the US, successful platforms like Android and iOS have many inframarginally multihoming developers. Those developers would not change much if the installed base of users were changed, so unless users react very strongly to changes in the supply of apps, platform equilibrium will be stable.

The economic intuition that stability arises from inframarginal multihoming leads us to consider other market structures as well. One is market like that of the current US, but with much smaller demand. Would a platform equilibrium with the same shares – but much smaller installed bases of users – still satisfy our stability bound? A similar, but more subtle question concerns a market the same size as the current US, but with a significantly larger market share for one of the smaller platforms, e.g. Windows Mobile, than it has enjoyed historically. If such a market would not satisfy our stability bound, that would suggest that the smaller platforms have “tipped out.”

V ECONOMETRICS OF DEMAND AND DEVELOPER SUPPLY

We estimate the parameters of user demand for apps on each platform, developer signals about profitability, and the decision rule determining supply of an app to each platform. Our dependent variables are S , S^* , and r^* . In this section we lay out our econometric model of demand and supply and our integrated model of selection of potential entrants.

V.A App Demand specification

We model users' demand for apps but not users' choice of platform.²² Demand for app a on platform p is $r_{pa} U_p$. We model the distribution of *per-user* demand r_{pa} . We choose functional forms so that $0 \leq r_{pa} \leq 1$.

21. See appendix for details. In our empirical specification $\hat{r}_p$ varies with regressors X and v_p depends on λ as well as $\hat{r}$. These complications are addressed in the appendix.

22. We thus cannot test hypotheses about user preferences for devices. A large commercial literature reports that very different users chose iPhone, most notably richer ones (see discussions in Bresnahan, Davis, & Yin (2015)).

Following our discussion of industry institutions, we will distinguish between *potential reach*, per-user demand if all users were aware of the app, and *realized reach*, per user demand.

The distribution of *potential reach* $\tilde{r}_a = (\tilde{r}_{ia}, \tilde{r}_{da})$ has a marginal *beta* distribution on each platform and can be dependent across platforms. To permit dependence, we build a mixture model from three underlying independent *beta* distributions called q_i , q_d and q_b . Potential reach is either equal to the independent pair (q_i, q_d) or to the perfectly dependent (q_b, q_b) .²³ Here, $q_i \sim \text{beta}(\alpha_{iX}, \beta_{iX})$ and Android demand $q_d \sim \text{beta}(\alpha_{dX}, \beta_{dX})$. The notation param_{pX} means that param for platform p is a regression on X . We restrict the distribution of q_b to be *beta* and to have mean and variance which are the average of those of q_i and q_d .²⁴ In some specifications we permit ω , the dependence parameter, to vary with observables and write ω_X .

This parameterization lets us pursue several of our research goals. We can study symmetry of demand across platforms by permitting the parameters to vary with p . We always let α_{dX} differ from α_{iX} and in some specifications β_{dX} need not equal β_{iX} . To sharpen the inference of demand symmetry, we can divide the dependence in app demand across platforms into unobserved (ω) and observed (projected on X) components. We can test whether users of the two different platforms tend to demand the same kind of app as measured by X .

We now turn to the distribution of *realized reach*. We assume that $\bar{r}_p$, is a “shrunk” version of $\tilde{r}_p$, i.e. (again suppressing dependence on X for a moment) that $\bar{r}_p \sim \text{beta}$ and that $E[\bar{r}_p] = \delta E[\tilde{r}_p]$ and $\text{var}[\bar{r}_p] = \delta \text{var}[\tilde{r}_p]$, with $\delta_p X_a$ parameters to be estimated.

We model the realized demand for the app, r_p , as

$$(4) \quad r_p = \begin{cases} \tilde{r}_p & \text{with probability } \lambda \\ \bar{r}_p & \text{otherwise} \end{cases}$$

V.B Developer Supply Specification

A developer writes for platform p if $E[r_p U_p M_p | \tilde{r}_p] > C_p$ (see equation (1)). We simplify the supply parameters, estimating only the entry thresholds κ_p , where $\kappa_p = C_p / (M_p \times U_p)$. Without a variable like price (most apps are free) we cannot separately identify M vs. C , only their ratio. The role of U_p means that this is also a normalization. $\kappa_d = \kappa_i$ corresponds to symmetric supply given the observed market sizes (U_d and U_i), but it does not mean that C_p / M_p is the same across platforms. In some specifications we let

23. There are several other ways to model dependent *beta* distributions. The Sarmonov method is inappropriate for our purposes, since it limits the correlation to be near zero. Another method is to build up the distributions from ratios of *gamma* distributions. This, however, does not lead to both marginal and conditional *beta* distributions and thus would leave a number of the calculations below much more difficult.

24. That is, we impose $\mu_{bX} = \frac{\mu_{iX} + \mu_{dX}}{2}$ and $\sigma_{bX}^2 = \frac{\sigma_{iX}^2 + \sigma_{dX}^2}{2}$ for each X and then solve $\mu_{bX} \equiv \frac{\alpha_{bX}}{\alpha_{bX} + \beta_{bX}}$ and $\sigma_{bX}^2 \equiv \frac{\alpha_{bX}\beta_{bX}}{(\alpha_{bX} + \beta_{bX})^2(\alpha_{bX} + \beta_{bX} + 1)}$ for the parameters $(\alpha_{bX}, \beta_{bX})$.

κ_{pX} vary with X .

These assumptions mean app a is written for platform p (dropping the X) if:

$$(5) \quad \mathbb{E}[r_p | \tilde{r}_i, \tilde{r}_d] \geq \kappa_p \iff \{\lambda \tilde{r}_p + (1 - \lambda) \mathbb{E}[\tilde{r}_p | \tilde{r}]\} \geq \kappa_p,$$

This gives us the distribution of the bivariate supply dummy S . The events determining the three observable cases are:

S_i	S_d	<i>Event</i>	
0	1	$(\kappa_i - \lambda \mathbb{E}[\tilde{r}_i X, \theta]) / \lambda \geq \tilde{r}_i$	$\& \quad (\kappa_d - \lambda \mathbb{E}[\tilde{r}_d X, \theta]) / \lambda < \tilde{r}_d$
1	0	$(\kappa_i - \lambda \mathbb{E}[\tilde{r}_i X, \theta]) / \lambda < \tilde{r}_i$	$\& \quad (\kappa_d - \lambda \mathbb{E}[\tilde{r}_d X, \theta]) / \lambda \geq \tilde{r}_d$
1	1	$(\kappa_i - \lambda \mathbb{E}[\tilde{r}_i X, \theta]) / \lambda < \tilde{r}_i$	$\& \quad (\kappa_d - \lambda \mathbb{E}[\tilde{r}_d X, \theta]) / \lambda < \tilde{r}_d$

We call the probability of each of these events $Pr(S|X, \theta)$ and we can calculate this probability easily as a function of parameters and X given our assumption that the joint distribution of $\tilde{r}$ is a mixture of *betas*. See appendix equation XXX.

V.C Calculation of Likelihood and Sampling correction

The joint distribution of r^* , S^* and S given X and parameters θ is denoted $f_Y(S, S^*, r^* | X, \theta)$. To write this out for an individual observation (dropping the a subscript for clarity) we first note that there are 5 possible observed values of S, S^* for sampled observations, i.e. for observations with $S_i^* = S_d^* > 0$.

S_i	S_d	S_i^*	S_d^*
0	1	0	1
1	0	1	0
1	1	1	0
1	1	0	1
1	1	1	1

For each cell, we write $f_Y()$ as

$$f_Y(S, S^*, r^* | X, \theta) = Pr(S|X, \theta) * Pr(S^*|S, X, \theta) * f(r^*|S, S^*, X, \theta)$$

Above, we showed how we calculate $Pr(S|X, \theta)$ based on Equation (5). Conditional on S , we calculate the mixture distribution of $\tilde{r}$ using the underlying *beta* distributions and (4).

We then work out the sampling distribution of S^* and of r^* conditional on S and $\tilde{r}$. On each platform, comScore has a sample of 5,000 users. Let g_p be the number of platform p users that have the app in the

comScore sample. The density of g_p conditional on r_p is binomial, and r_p^* is simply $\frac{g_p}{5,000}$. The distribution of r_p is a *beta* mixture, so the distribution of r_p^* is a *beta – binomial* mixture. Lastly, comScore’s reporting criterion dictates that $S_p^* = 1$ whenever $r_p^* \geq \frac{6}{5000}$. Given the *beta – binomial* structure of r_p^* , the probability of this event can be calculated in closed form.

Our sample of apps is selected: we only observe apps which meet comScore’s sampling criterion on at least one platform, i.e. apps for which $S_{ia}^* + S_{da}^* > 0$. The structure of the joint distribution of r^* , S^* and S permits us to calculate the probability of this event, denoted $\Pr(S_{ia}^* + S_{da}^* > 0 | X_a, \theta)$ in closed form. We correct for sample selection by dividing by this probability and maximizing the conditional likelihood:

$$L_C(S, S^*, r^* | X, \theta) = \sum_a \log\left(\frac{f_Y(S_a, S_a^*, r_a^* | X_a, \theta)}{\Pr(S_{ia}^* + S_{da}^* > 0 | X_a, \theta)}\right)$$

This solves the long-standing “potential entrants” problem in entry models. The problem arises in all analyses which, like Berry (1992), identify a list of potential entrants into markets. Typically, the list of potential entrants into one market is constructed from actual entrants into another market.²⁵ If – as one would nearly always suspect – a firm’s profit in one market is not independent of its profit in another, the list of potential entrants so defined is not exogenous but instead selected. This problem certainly applies to our application; we list actual entrants into platform p as the potential entrants into platform p' . We need a modeling solution; no dataset could be constructed containing all the developers who considered launching a mass-market app on a particular platform.

Typically, a model of selection adds parameters to be estimated and additional modeling assumptions. Our model does not. We build economic and econometric models of entry into both markets. This creates not only a model of supply, our goal, but also a model of selection, as entry into at least one of the markets is how a firm goes on the list of potential entrants into the other.

VI RESULTS

Estimated parameters are presented in Tables 3 (parameters that vary with p) and 4. Bootstrapped confidence intervals are presented below or beside each parameter estimate.

VI.A App Demand on iOS and on Android

Our app demand specification permits demand for a particular app to vary with platform p and with observable features of the product and the supplier X . The baseline constant is “Online.” In our preferred

²⁵ The market entry literature is reviewed in Berry & Reiss (2007). Some papers like Bresnahan & Reiss (1991), Mazzeo (2002), and Seim (2006) identify a set of market niches rather than a set of potential entrants and thereby avoid the selection problem.

TABLE 3: PARAMETER ESTIMATES PART 1

	α_d	α_i
Constant	0.3124** (0.1035)	0.3003** (0.0984)
Offline	-0.0083 (0.0710)	-0.0000 (0.0795)
Mobile Only (MO)	-0.1195 (0.1175)	-0.2005* (0.1088)
Game	-0.0781** (0.0328)	-0.0224 (0.0252)
Publ. Traded (PT)	0.0670 (0.0550)	0.1485** (0.0672)

Note: Bootstrapped standard errors (250 draws) below each coefficient. **significant at 5% *significant at 10%

TABLE 4: PARAMETER ESTIMATES PART 2

	Estimate	Standard Error
κ_d	0.0011**	(0.0004)
κ_i	0.0011**	(0.0004)
β	22.5658**	(0.4536)
λ (constant)	0.6480**	(0.0728)
λ (MO)	-0.0983	(0.1158)
ω	0.3482**	(0.0631)

Note: Bootstrapped standard errors (250 draws). **significant at 5%

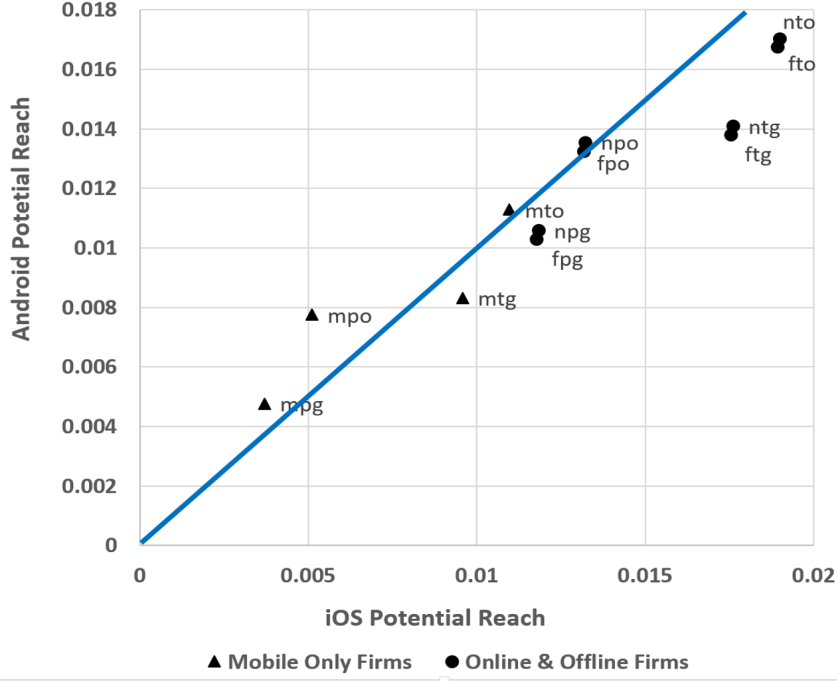
specification, app demand on platform p is distributed *beta* with parameters α_{Xp}, β .²⁶ In this specification, only the α_{Xp} vary across observable type X and platform p . We set $\delta = 0.02$, consistent with findings in Bresnahan, Li, & Yin (2015) regarding the difficulty of getting visibility on a top list in the app stores.

The first row of Table 3 indicates that $\alpha_i \approx \alpha_d$ for apps by firms who already have an Online established presence, so the variation in demand for these apps is similar across the two platforms. We cannot statistically reject the equality of those alphas. The apps by firms with an Offline established presence seem to be statistically no different than the online firms. However, Mobile Only firms have a lower alpha (and subsequently will have a lower reach) than the established firms on iOS. Though also lower, the alpha for Mobile Only firms on Android is not statistically lower than that of the Online firms on Android. However, being a game on Android will lower the value of alpha, and Mobile Only Games comprise a large category of apps. Finally, being a Publicly Traded firm on iOS will increase alpha.

All of our X are dummies. There are 12 distinct values of X . In Figure 4, we show average demand (potential reach) for each Xp combination, calculated as $\alpha_{Xp}/(\alpha_{Xp} + \beta)$. Each point is labeled by X . The points all lie approximately along the 45° line, though they are spread out along it. Even for the

26. We also have a more richly parameterized specification in which β_{Xp} also vary. Little differs in that specification, so we do not present those results here.

FIGURE 4: JOINT POTENTIAL REACH ON IOS AND ANDROID BY APP-DEVELOPER TYPE



Type abbreviations with number of observations in parentheses: mpg=mobile private game (177), mpo=mobile private other (257), mtg=mobile traded game (3), mpog=mobile traded other (1), npg=online private game (48), fpg=offline private game (22), npo=online private other (151), fpo=offline private other (76), ntg=online traded game (24), ftg=offline traded game (53), nto=online traded other (80), fto=offline traded other (152).

apps exhibiting the most asymmetry between platforms, the publicly traded established apps, the difference between their reaches is only 0.3% of the population. From this, we conclude that mean app demand does not vary across platforms, but does vary with X .

Our model also permits unobserved dependence across platforms in users' demand for a particular app conditional on X through the parameter ω . We find some unobservable dependence on top of the observable dependence, as our point estimate of $\omega = 0.35$.

Our emphasis here is on the differences across platforms and their relationship to platform equilibrium. The conclusion about demand is quite simple. We have strong app demand symmetry. Demand varies widely across apps, to be sure, but there is strong positive dependence between the values of $\tilde{r}_i$ and $\tilde{r}_d$.

VI.B Supply for iOS vs. Android.

Supply parameters (κ) are presented in Table 4. Our supply equation is $S_p = 1 \Leftrightarrow \tilde{r}_p > \kappa_p$, so these estimates imply developer supply symmetry. Developers' thresholds for writing on iOS (.0011) are the same as the threshold for writing on Android. In round numbers, a developer supplies to either platform if forecasted demand exceeds about 0.001 of users.

Because there are more Android users than iOS users, symmetry in κ implies a difference across platforms in the ratio of per-customer profit to fixed marketing and development cost, M_p/C_p . Since $\kappa_p \equiv C_p/(M_p \times U_p)$ and $U_i/U_d = 0.4/0.52$, our estimates imply $M_d/C_d = 0.769M_i/C_i$. That is, [per customer profit]/[fixed costs] is about 80% as large on Android as on iOS. While our estimates cannot distinguish between C and M , developers tell us that fixed costs of writing for each platform are approximately equal, while per-customer profits are higher on iOS, since iOS users tend to be richer. Our estimates are consistent with this external fact, pointing to an estimate of Android customers just over 80% as profitable as iOS ones.

Since the actual US market has slightly more Android users, and since we estimate $\kappa_i = \kappa_d$, we have strong evidence for existence of an equilibrium in which developers supply symmetrically across platforms. The same estimates show that a developer whose expected reach is the same on both platforms will multihome.

VI.C Existence of Symmetric Equilibrium

At this point, we have established that both app demand and app supply are approximately symmetric. These are two of the sufficient conditions for the existence of a fragmented equilibrium with approximately equally many users and apps on each platform. Our explanation is only partial because we have not estimated a user platform-demand model. Over a long period of time during which (a) about the same apps are available on iOS as Android, (b) iPhones are more expensive and less varied than Android phones, and (c) the best Android phones are similar in quality to iPhones, we do observe that user demand stays close to the 40%/52% ratio. It seems safe to assume that this is a point on the user platform demand curve.

VI.D Inframarginal Multihoming and Stable Equilibrium

We now turn to stability and the related topic of developer inframarginal multihoming.

First, we report our estimate of the stability bound on η_i^i . This is the smallest value of the elasticity of demand by users for iPhones with respect to v_i , the per user attractiveness of available apps, that will lead to instability. The precise formula is in the appendix. Recall, however, that we are going to model demand as a single-parameter logit and bound the parameter of that logit using our estimates of the parameters of app demand and supply.

In the first row of Table 5, we report this value for the historical duopoly in the US. The bound is enormous – equilibrium would be unstable only in the case where an increase by 1% in the attractiveness of iPhone apps would lead to a 40% increase in the demand for iPhones. The current, fragmented equilibrium will be stable unless users are *extremely* responsive to apps in their choice of platform. We get this very high bound because, in our estimates, developer supply (of v_p) is unresponsive to the size of user demand at the

TABLE 5: EMPIRICAL BOUND ON η_i^i

Historical Duopoly	38.64
Bootstrap Standard error	(7.85)
Duopoly with U halved	20.71
Bootstrap Standard error	(4.05)
Triopoly with Windows Mobile at 8%	6.37
Bootstrap Standard error	(2.73)

TABLE 6: INFRAMARGINALITY AND MULTIHOMING

	Platform Choices (S)			Weighted by User Demand (v_p)		Weighted by User Demand and Multihoming (v_p^{MH})	
	S_d	S_i	S_b	v_d	v_i	v_d^{MH}	v_i^{MH}
Baseline	0.555	0.534	0.402	28.265	26.419	19.724	19.997
U_d halved	0.468	0.534	0.344	27.691	26.419	19.380	18.529
U_i Halved	0.555	0.454	0.344	28.265	25.904	18.309	19.639
Both Halved	0.468	0.454	0.320	27.691	25.904	18.191	18.399

observed market shares.

In the second row of the same table, we report this value for the same market but where the population of users has been halved. We find that the bound is still quite large, but about half that of the full market. Reducing the market size makes the duopoly less stable. We now investigate the origins of that estimate in the details of our estimates of developer supply.

Most platform supply behavior, including most multihoming behavior and most provision of apps that meet high demand, is *inframarginal* according to our estimates. In Table 6 we report counterfactuals in which we change market sizes, i.e., in which we change the number of users on each platform. The columns of this table are statistics of developer supply, S_p , v_p , and the part of v_p that comes from multihoming apps, v_p^{MH} , defined as $\int_r r_p S_b(r) f(r) dr$. We have $v_p^{MH} < v_p$ since the former excludes the contribution of single-homers.

In the first counterfactual, we cut U_d in half. This causes supply to the Android platform to decrease by about 16% (from 0.555 to 0.468) and has an even smaller impact on multihoming (about 14%). Symmetrically, halving U_i leads to the same changes in S_i and S_b . These results are determined by the shape of the estimated distribution of app demand, which depends on the distribution of X as well as on the shape we assumed for $f(r)$. Given our model and our estimates, we conclude that most observed supply decisions are inframarginal, since no more than 9% of app supply ever changes on a platform, even when both platforms

are halved.

The implication of our estimates for v_p are even more stark. In the last four columns of Table 6, we report first the predicted and then the counterfactual (with changing U_p) values of v_p and v_p^{MH} . Comparing the multihoming columns with the columns containing estimated user value from all apps, we see that the value of multihomers is a large proportion of app value overall. Halving user demand for either platform does not change the user value from all apps by more than 2%, suggesting that marginal apps do not have much value. The effect of reductions in users on the value of multihomed apps is greater (although only a drop of about 7% for either platform), but note that the effect is largest when users are cut on the *opposite* platform. The decline in value is nearly non-existent when the decline is on one’s own platform. This is indicative of skewed value of the inframarginal multihomers. When supply drops on Android due to a drop in users on Android, 3% of the apps are single homers who no longer supply, and 6% are multihomers who no longer supply to Android, yet consumer value from inframarginal multihomers remains largely unchanged. However, when the other platform’s users drop, supply to Android remains the same. That means that all the multihomers switched to being single homers on Android. Total app value does not change on Android, but there are fewer multihomers, and therefore there must be a lower value for the multihomed apps on Android.

It is not clear that our results about inframarginal multihomers would extend to much less popular apps. Our sample selects developers based on the low threshold that they gain one in a thousand potential customers on at least one platform. Thus we cannot learn much about the demand for, or behavior of, developers under that threshold. For our purposes, this is not problematic. In a mass-market business, the contribution to app attractiveness of those apps too unpopular to be in our sample can safely be neglected.

Examining both developer decisions and user value in response to changes in the distribution number of users across platforms, we find that the fragmented equilibrium is stable. The importance in user demand of apps from developers who are inframarginal multihomers means that there is little incentive for users or developers to change their behavior should equilibrium be perturbed.

One other, non-historical, market structure can deepen our understanding about what our model is saying about platform market equilibrium. The central feature of this counterfactual is to consider developer supply to an installed base of devices that is far smaller than that relevant to the historical US experience of Android and iOS.

In the third row of 5, we report the threshold demand elasticity (η_i^i) for a market structure that is closely related to the US duopoly. We add a third platform, and endow it with a market share of 8%, keeping the installed base of Android and iOS fixed at historical levels. Roughly speaking, this corresponds to a market structure in which a third platform has the combined historical market shares of both Windows Mobile and

Blackberry. We have no estimates of the app demand or supply for such a platform. So we endow the counterfactual platform with the app demand and app supply parameters we have estimated for Android. (Similar results arise when we use our iOS estimates instead.) Now the stability bound is calculated from the eigenvalues of a three-by-three jacobian. Nonetheless, we bound η_i^j at a very similar point as in the first row, as the installed base of iPhones is the same as in the historical world.

This leads to a dramatically lower stability bound than in the historical world. The difference here is that there is a platform with a significantly smaller installed base, so that many fewer developers supply it inframarginally. The parameter estimates are the same – what has changed is that our third platform has a much smaller installed base. This counterfactual underscores the possibility that the two smaller platforms ”tipped out” in the US.

VI.E Entrepreneurship and the Established

As we pointed out in our Industry Section II, many observers anticipate that mobile app development will be an entrepreneurial business because of the low technical costs of entry. In earlier work, based on an examination of app developers’ monetization strategies (Bresnahan, Davis, & Yin (2015)) we advanced the contrary hypothesis that lack of consumer familiarity of entrepreneurial firms’ brands and marketing costs would disadvantage entrepreneurial firms. The latter hypothesis appears to be the one borne out in our demand estimates. Returning to Figure 4, we see that the higher-demand developer categories tend to be established (online and offline) firms. This follows from our estimates of α in Table 3. We have $\alpha_{MO} < 0$ and $\alpha_{PT} > 0$. This may reflect demand preference differences between apps offered by entrepreneurial, Mobile Only firms and established firms (Online and Offline, Publicly Traded firms). This may also reflect differences faced by entrepreneurial and established firms in the marketing costs to reach customers. In our point estimates, it reflects both of these. Our estimates corroborate our industry observations that the more established firms use existing customer relationships as a substitute for the marketing efforts required for entrepreneurial firms, and therefore could more easily reach more customers. The costs of the app store fall heavily on firms that are trying to acquire new customers in the mobile world (e.g. mobile entrepreneurs) and much less for those for whom the mobile world is an extension of existing customer relationships (e.g. banks). Regardless of the particular driver, the incentives to multihome are much greater for established firms due to higher realized reach than for entrepreneurial firms. These asymmetric reaches and thus incentives for multihoming indicate why established firms, rather than entrepreneurial firms, have played a much larger role in this industry than anticipated.

There is one conclusion from all these perspectives: mobile apps vary widely in their ability to reach

consumers, whether due to preferences or costs of cutting through the app store, but the same mobile app has a strong tendency to have about the same reach to consumers on both platforms. The apps with the highest reach have the largest incentive to multihome, and these apps tend to be from established, not entrepreneurial, firms.

VII IDENTIFICATION

We now turn to identification, and focus first on the key economic quanta needed for existence of fragmented equilibrium and for the bound on stability. Both of these are, we show, in the data rather than artifacts of specification. First, the symmetry of app supply and demand that we estimate is in the data, not a modeling artifact. We shall spend the next two subsections discussing this in detail, but our argument turns on the point that all the moments relevant to supply, demand, or selection are symmetric across platforms. Second, our specific quantitative results about inframarginal app supply and stability are of course dependent on our functional form assumptions. However, our result that the dominant supply behavior among the developers whose apps are most demanded is inframarginal multihoming has a strong foundation in the data, which we shall exhibit.

We will *not* make an effort to show that all our parameters are well identified, for this is not necessary to our economic argument.²⁷ We will also only show identification based on observing a fragmented platform market structure, the data we have. As with platform tipping papers, generally, identification at other points, such as a single dominant platform, is likely to fail.

VII.A *Symmetry of App Demand*

By symmetry of app demand across platforms, we mean that users on one platform tend to value the same apps as do users on another. Our estimates of app demand – which of course depend on all our functional form assumptions, not just on the data, are symmetric. We saw above that point estimates in Figure 4 lie approximately along the 45° line. These inferences of demand symmetry follow directly from the data.

First, we have already seen some evidence symmetry in app demand across platforms in the observed symmetry of the distribution of r_d^* , r_i^* in Table 1 and, for apps observed on both platforms, in Figure 2. The marginal moments of per-user demand are the same on both platforms and there is tremendous positive dependence in demand on each platform in the cross section of apps.

It is easy to see that the distributions of r^* are very similar on both platforms conditional on our X

27. Thus we cannot use the techniques usefully brought forward in Gentzkow and Shapiro (2015), who propose a structured method for analyzing parameter identification.

TABLE 7: MEANS OF OBSERVED REACH CONDITIONAL ON X

X	$X=1$		$X=0$	
	r_d^*	r_i^*	r_d^*	r_i^*
mobile only	0.0134	0.0131	0.0221	0.0245
offline	0.0154	0.0161	0.0196	0.0237
online	0.0134	0.0151	0.0233	0.0257
game	0.0103	0.0134	0.0214	0.0241

variables as well. In Table 7, we show mean reach for observed apps conditional on several of our X variables taking on the value 1 and also conditional on $X = 0$. For Mobile Only, for example, we see that both r_d^* and r_i^* have means around .013 for mobile only firms, and means of .022 and .0245 for all other firms. Similarly, Offline firms have lower mean reach than other firms on both platforms, and online firms have higher mean reach on both platforms. Our inference of user app demand symmetry reflects this underlying symmetry across platforms in app demand data.

It is not possible to perform a completely non-parametric selection correction in our context. We cannot, for example, rule out some asymmetries in demand across platforms, such as the possibility that the distribution of r_d and the distribution of r_i are very different over the range not too far from $r > .001$ that we see. However, it is the upper tail of the distribution which matters for the bulk of usage, and all indicia are that the lower tail is not important.

For any parametric model, it is very difficult to see how selection would change the inference of symmetry, and certainly it does not in our model. The selection rule has two parts, (1) comScore’s rule for reporting reach, which is symmetric across platforms and (2) developer’s supply behavior, which of course could in principle be symmetric or not.

We know from Table 2 that observed developer app supply behavior is approximately symmetric. Just under two thirds of developers multihome, a very symmetric supply behavior. Only slightly more developers supply for Android (0.82) as for iOS (0.77). When we condition on X , we continue to find symmetry in observed developer behavior. Table 8 shows similar rates of supplying for d and supplying for i as X changes.

Symmetry in observed supply behavior is relevant to any parametric selection correction. Whether one uses the symmetry of the ComScore rule or not (we do), a selection correction can change the estimated moments of demand. However, given the symmetry of the observed supply behavior across platforms, a selection correction that is applied symmetrically to both platforms will change those moments symmetrically.

TABLE 8: MEANS OF S CONDITIONAL ON X

X	X=1			X=0		
	S_d	S_i	S_b	S_d	S_i	S_b
mobile only	0.8699	0.7831	0.6530	0.8531	0.7937	0.6469
offline	0.8680	0.7921	0.6601	0.8570	0.7881	0.6451
online	0.8657	0.7711	0.6368	0.8532	0.7945	0.6478
game	0.8685	0.7859	0.6544	0.8563	0.7908	0.6471

TABLE 9: MEANS OF S^* CONDITIONAL ON X

X	X=1			X=0		
	S_d^*	S_i^*	S_b^*	S_d^*	S_i^*	S_b^*
mobile only	0.5799	0.6530	0.2329	0.5693	0.6601	0.2294
offline	0.5545	0.6601	0.2145	0.5816	0.6559	0.2375
online	0.5771	0.6368	0.2139	0.5765	0.6625	0.2390
game	0.5505	0.7003	0.2508	0.5844	0.6374	0.2218

VII.B Identification of Inframarginality

Our parametric conclusion was that most large developers are inframarginal multihomers. We now consider what information in the data has led to this conclusion.

In Figure 2, many realizations of both r_d and r_i lie far up and to the right. Any model, not just our model, with rational expectations by developers is going to make the prediction that multihoming will be frequent for large developers.

Our inference that supply decisions serving most of user app demand are inframarginal is, of course, based on our model in which there is an entry threshold. Conditional on that assumption, our conclusion appears to be clearly in the data. We show the distribution of the observed reach data in Figure 5. Bins are labeled by their upper boundary, so the top bin is labeled “1”. The log scale means that each bin covers $[y, 2y]$ for some y . Since we only observe r^* if it is over .001 the lowest bin with a numerical name is .002. Below that, we have a bin labeled “Not Observed,” which corresponds to apps that were written but do not appear in comScore. The lowest bin is apps which are in our sample but not written for the platform.

The point of this figure is that the bulk of demand – r – is coming from apps which are above the .004 bin. Not necessarily the bulk of apps, but the bulk of app demand.

There are two ways in which this argument is incomplete, and we do not have a specification-free way to complete it. First, the figure is based on our selected sample. Any reweighting of the figure to a population basis would be based on parametric estimates. We are untroubled by this, however. A sample selection correction that did not rely on our parametric form would, if it were to change the inference about

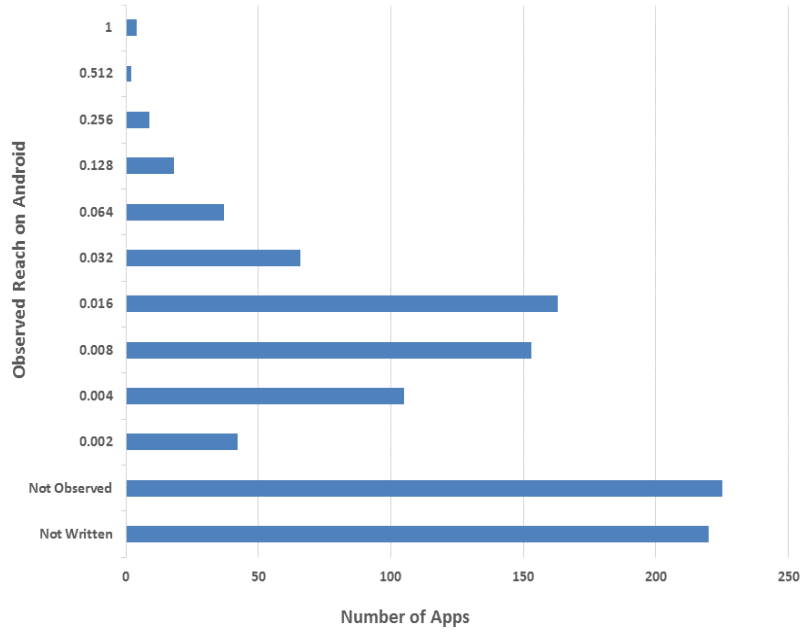
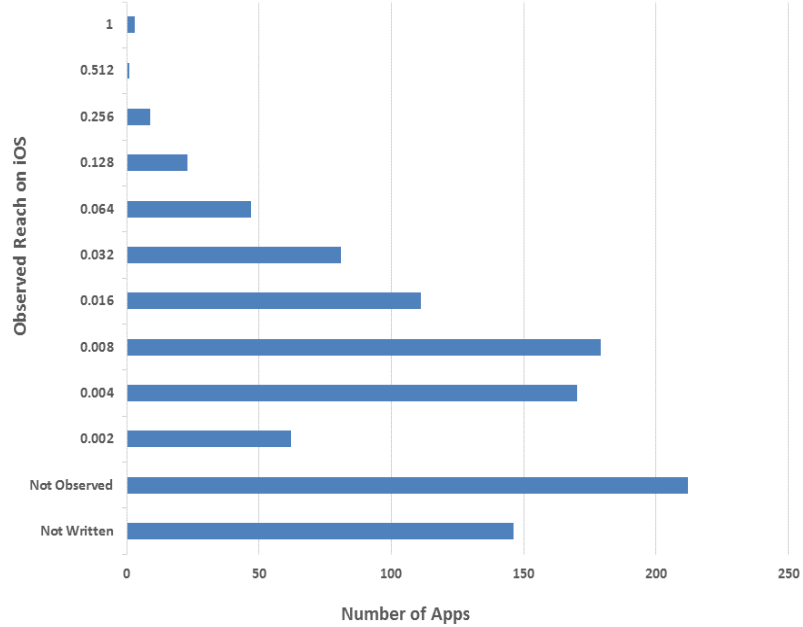
inframarginal supply, need to change the inference about the density of app demand at high demand levels (10s of millions of users.) That would be a very gross error wrought by our parametric form – the true model would have to posit that there are many 10-million user apps on platform p that we are missing because the apps were not written on platform p' only because we imposed our parametric structure.

The second possible problem with interpreting Figure 5 is the structure of our entry threshold model. We assume that the app is written for the platform is a signal of expected demand (reach) is above a threshold, and that the threshold is the same for all developers with the same X . It is logically possible that firms have different fixed costs of writing the app, and that these fixed costs are positively correlated with demand. If that were true, then we would be incorrect to infer that apps farther up in Figure 5 tend to be systematically farther from the entry threshold. We cannot rule this out nonparametrically. The log scale of Figure 5, however, implies that there would need to be some extremely large fixed costs to drive that hypothetical positive correlation. Our engineering and management sources in the industry tell us, further, that the technical fixed costs are driven by the complexity of the app, not by its intended user base, and that the marketing fixed costs are, if anything *decreasing* in demand since popular apps get free advertising from the app store top lists.

The inference that platform supply decisions, especially those most important to v , are not close to the margin in our model thus seems to be based largely on the data. The distribution displayed in Figure 5 shows that many entrants have very large realized demand. It is this feature of the data that leads our model to calculate that a large change in U_p would have little impact on these firms' behavior.

Our other finding that S (and not just v) is not very responsive to changes in U_p relies more on our parametric structure. For this conclusion, the firms in the bottom two bins of Figure 5 play a larger role. Simply examining the figure one sees that the sample selection rule matters for the predicted density near the entry threshold and the possibility that the parametric form of the selection rule matters. The many firms who supply and do not have a serious prospect of mass market acceptance fall outside our sample entirely, and we cannot say anything about them.

FIGURE 5: SIZE DISTRIBUTION OF OBSERVED REACH (LOG SCALE) r_i^* AND r_d^*



VIII CONCLUSION

The platform race on smartphones, central to one of today’s most important technologies, has been in a stable equilibrium with approximately equal market shares for iOS (iPhone) and Android phones in the US for several years. We investigate a new explanation for the stability not found in the previous literature, rooted in the industrial organization of the supply of mobile apps, a complement to mobile platforms. In our explanation, the equilibrium supply of apps to platforms involves *multihoming* by most of the apps that are strongly attractive to users. User platform choice – among the leading platforms – is thus not heavily influenced, in equilibrium, by app availability; most of the attractive apps are available to users whichever of the leading platforms they choose. Our explanation of stability requires more. If most app developers’ platform supply choices, including their decision to multihome, are *inframarginal* at an equilibrium, the equilibrium will tend to be locally stable. This follows because local changes in the number of users choosing each of the leading platforms will not lead to substantial changes in app developers’ platform supply decisions.

Both existence and stability of such an equilibrium depends on underlying supply and demand conditions. We proceed in two steps to verify them empirically. First, we have identified a set of positive predictions of our explanation of existence and stability of an approximately-split equilibrium. We test these using our model, and in some cases simply by examining moments of the raw mobile apps data. The important demand conditions concern dependence across the (leading) platforms. If an app is strongly attractive to users on platform p , it should also be highly attractive to users on platform p' . This demand condition, together with the supply condition that app costs and per-user profits are also approximately symmetric across platforms, will lead to multihoming in app supply when there are sufficient numbers of users on each leading platform. The most important additional supply conditions for stability are based on the size distribution of apps. Even if attractive app supply is highly concentrated, if this concentration is so much that most suppliers are far above minimum efficient scale for each platform, then most supply decisions will be inframarginal. We have investigated all these conditions, and all hold for the two leading smartphone platforms for mobile apps in the US, iOS (iPhone) and Android.

As a second step, we derive the (conventional) local stability condition at equilibrium. As in all platform economics research investigating tipping or its absence, there is an elasticity we cannot estimate. In our case, it is users’ elasticity of substitution between iOS (iPhone) and Android phones. We then predict the changes in supply that would result from changes in the number of users on the platform. We find that the supply of apps and value to consumers of those apps remains quite stable in the face of large changes to the number of users from current levels. Since a number of important shocks to mobile phone supply, such as the introduction of new iPhones and of new, highly attractive Android phones, have not led to extreme

swings in platform market shares, we feel more confident in our stability inferences.

We can sharpen our argument by noting that, under our explanation, the conditions for stability apply only to the leading platforms in a large market like the US. Our conditions absolutely do not mean that the platform race on smartphones is always and everywhere not “tippy.” First, our explanation applies only to approximate ties between leading platforms, and implies that equilibrium forces tend to decrease the share of a platform that is *far behind*. The critical difference between a platform that is far behind and a leading platform is related to the crucial role of inframarginal developer multihoming decisions. The decision to supply an app to a platform with far fewer users than a leading platform, even if undertaken, will *not* be far from the margin.

This explains the parts of the mobile platform race which have in fact proved “tippy.” First, we examine the fate of platforms which have tipped out in the US – Windows Phone and Blackberry. Our evidence that they have tipped out is less quantitative than our evidence concerning the leading platforms; it is prohibitively hard to econometrically estimate the demand for Windows Phones or Blackberries if they were to have adequate app supply. Similarly, we note that in other countries with smaller markets or not enough rich users to sustain the demand for iPhones, iOS has tended to tip out. This, too, is entirely consistent with our explanation. Fewer multihoming supply decisions will be inframarginal in a smaller market.

The industrial organization of the supply of apps is crucial to our explanation of platform market equilibrium and stability. Our estimates show that apps from established firms have much higher expected demand than apps from mobile entrepreneurs. These include established consumer-oriented online firms such as Facebook, Yahoo!, and Google, and established consumer products and services firms, such as banks and airlines. Consideration of the strategic situation of these firms makes it obvious why our model produces the result that most of them are inframarginal multihomers in their mobile apps platform choice. These are mass market consumer firms, and with the exception of Google, mobile apps are a complement to their main business, a way to extend their customer connection into the mobile world. These firms tend to have many customers, and their customers tend to pick smartphones for reasons not particularly related to, e.g., their airline ticket app. It is little surprise that they have incentive to multihome to platforms used by a significant fraction of their customers, such as the two leading platforms in the US. For the online subset (e.g., Facebook), scale economies and social network effects around their established line of business further drive them to multihome to build their user base.

Finally, the industrial organization of complements is an important differentiator of this industry from others. Our industry differs from the contemporary (post-1995) PC industry, which has tipped to a Windows standard, in the size distribution of applications and in the industry structure of other complements. In the PC industry, only a few applications, such as Word, Excel, and PowerPoint, are very widely used, and

those are supplied by one platform provider. Other apps, typically with significantly smaller reach, don't multihome or delay multihoming, so that app prevalence is an important part of user platform demand. In an earlier phase of the PC industry (1975–1994) movements from one platform control to another were frequent; in this era, there was vertical disintegration between platform providers (e.g. IBM, in the IBM PC) and key complementors (e.g. Microsoft, Intel) leading to divided technical leadership. Like the more recent PC industry, the older PC industry exhibits a very different role for the industrial organization of complement supply than we see in the current smartphone apps platforms. Platform industry equilibrium is correspondingly different.

Another industry that is similar in many ways is game consoles – indeed, many of the apps we study are games. But it is the industrial organization of supply in the complementary goods industry, not the content of the complementary goods, which is relevant to platform market equilibrium analytically.²⁸ Game console platform providers have far closer contractual relationships with app developers than do mobile platform providers, including sometimes having exclusive contracts with app providers. Game console platform markets have also been repeatedly tippy, with a dominant platform in each generation, but not the same dominant platform each time. These platform market outcomes reflect two important differences in the industrial organization of the key complement, in this case, games. The first critical difference is in the relationship between technical progress in the platform and in the complements. Game consoles of a new generation typically call for radical revisions for games: new generations of iOS and Android are overwhelmingly backward compatible, and the stock of apps for an old version is largely the same as the stock for a new. This gives platform providers in mobile game consoles repeated incentives to contract for exclusivity with “hit” games in order to tip the market in the next generation. In contrast, mobile platform providers have not sought exclusives. To be sure, one of the most important apps, Google Maps, is vertically integrated with Android's platform provider. So far, however, that relationship has not been used by Google to disadvantage Apple. Instead, Apple made an attempt to exclude Google Maps from its platform, reversing this very unpopular decision only after a user revolt. It seems, therefore, that the different relation between apps – even games – and the mobile platforms is another way in which this industry is unlike game consoles.

The observations of the last few paragraphs that our finding of a stable, balanced equilibrium in US mobile phone app development platforms because of a particular theory is not only consistent with the quantification of supply and demand of apps within that industry revealed in our model, but also with differences between that industry and other platform industries. The industrial organization of the supply of complements to a platform is an important determinant of platform market equilibrium.

28. Corts and Lederman (2009) study developer supply in the game console industry. Kretschmer and Claussen (2013) study handheld game consoles and emphasize very different platform economics than do we.

IX BIBLIOGRAPHY

- Armstrong, Mark. 2006. "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37(3), 668-91.
- Augereau, Angelique, Shane Greenstein, and Marc Rysman. 2006. "Coordination vs. Differentiation in a Standards War: 56K Modems." *The RAND Journal of Economics*, 37(4): 771-88.
- Bagnoli, Mark, and Ted Bergstrom. 2005. "Log-concave probability and its applications" *Economic Theory* August 2005, Volume 26, Issue 2, pp 445-469
- Bass, Dina and Adam Satariano - Jul 14, 2010. "Microsoft Pays Mobile App Developers to Help It Catch Apple", *Bloomberg*. <http://www.bloomberg.com/news/print/2010-07-14/microsoft-pays-developers-to-build-mobile-apps-to-help-catch-up-with-apple.html> accessed May 28, 2014.
- Berry, S.T. (1992), "Estimation of a Model of Entry in the Airline Industry", *Econometrica* 60(4): 889-917.
- Berry, Steven & Reiss, Peter, 2007."Empirical Models of Entry and Market Structure," in *Handbook of Industrial Organization*, vol. 3, Mark Armstrong and Robert Porter, eds. North-Holland Press.
- Berry, S.T., and E. Tamer (2006), "Identification in Models of Oligopoly Entry", *Econometric Society Monographs* 42, 46.
- Berry, S.T., and J. Waldfogel (1999), "Social Inefficiency in Radio Broadcasting", *Rand Journal of Economics*, 30(3): 397-420.
- Boudreau, K. (2010). *Open Platform Strategies and Innovation: Granting Access vs. Devolving Control*. *Management Science*. 56(10) 1849-1872.
- Boudreau, K. (2012). *Let a Thousand Flowers Bloom? An Early Look at Large Numbers of Software "Apps" Developers and Patterns of Innovation*. *Organization Science*.
- Boudreau, K. and A. Hagiu. 2009. *Platforms Rules: Multi-sided Platforms as Regulators*. A. Gawer, ed. *Platforms, Markets and Innovation*. Edward Elgar, London.
- Bresnahan, T., Davis, J. P., & Yin, P. (2015) "Economic Value Creation in Mobile Applications," *The Changing Frontier: Rethinking Science and Innovation Policy*, Adam Jaffe & Ben Jones, eds. (Chicago: University of Chicago Press)
- Bresnahan, Timothy F. and Shane Greenstein. 1999. Technological Competition and the Structure of the Computing Industry. *Journal of Industrial Economics*, 47(1): 1-40.
- Bresnahan, Timothy, and Shane Greenstein (2014) "Mobile Computing: The Next Platform Rivalry." *American Economic Review*, 104(5): 475-80.
- Bresnahan, T., Li, X., & Yin, P. (2015) "Paying Incumbents and Customers to Enter an Industry: Buying

Downloads,” mimeo.

Bresnahan, T.F. and P.C. Reiss (1990), “Entry in Monopoly Markets”, *Review of Economic Studies* 57: 57-81.

Brown, J. and J. Morgan (2009) ”How much is a Dollar Worth? Tipping versus Equilibrium Coexistence on Competing Online Auction Sites,” *Journal of Political Economy*, 117: 668-700.

Caillaud, Bernard and Bruno Jullien (2001) ”Competing Cybermediaries,” *European Economic Review*, 45, 797-808.

Caillaud, B. and B. Jullien (2003) ”Chicken and Egg: Competition among Intermediation Service Providers.” *RAND Journal of Economics*, 34(2), 521-552.

Cantillon, E. and P. Yin, ”Competition Between Exchanges: Lessons from the Battle of the Bund,” 2013
Casadesus-Masanell, Ramon, and Hanna Halaburda. ”When Does a Platform Create Value by Limiting Choice?” *Journal of Economics & Management Strategy* 23, no. 2 (Summer 2014): 259–293.

Church, Jeffrey, and Neil Gandal. 1992. ”Network Effects, Software Provision, and Standardization.” *Journal of Industrial Economics*, 40(1): 85-103.

Ciliberto, F., and E. Tamer (2009), “Market Structure and Multiple Equilibria in Airline Markets”, *Econometrica* Volume 77, Issue 6, pages 1791-1828, November 2009.

Corts, Ken, and Mara Lederman (2009) ”Software Exclusivity and Indirect Network Effects in the US Home Video Game Industry.” *International Journal of Industrial Organization*. 259-70.

Damiano, Ettore, and Hao Li. 2008. ”Competing Matchmaking.” *Journal of the European Economic Association*, 6(4):789-818.

Davis, J., Y. Muzyrya, and P. Yin, 2014. Experimentation Strategies and Entrepreneurial Innovation: Inherited Market Differences from the iPhone Ecosystem. Working Paper.

Davis, P. (2006), “Spatial Competition in Retail Markets: Movie Theaters”, *Rand Journal of Economics*. The *RAND Journal of Economics*, Vol. 37, No. 4 (Winter, 2006), pp. 964-982.

Dranove, D., & Gandal, N. (2003). The Dvd-vs.-Divx Standard War: Empirical Evidence of Network Effects and Preannouncement Effects. *Journal of Economics & Management Strategy*, 12(3), 363-386.

Dubé, Jean-Pierre, K. Sudhir, Andrew Ching, Gregory S. Crawford, Michaela Draganska, Jeremy T. Fox, Wesley Hartmann, Günter J. Hitsch, V. Brian Viard, Miguel Villas-Boas and Naufel Vilcassim (2005) ”Recent Advances in Structural Econometric Modeling: Dynamics, Product Positioning and Entry” *Marketing Letters* 16 (3-4), 209-224.

Ellison, Glenn and Drew Fudenberg. 2003. ”Knife-Edge or Plateau: When Do Market Models Tip?,” *Quarterly Journal of Economics*, 118(4), 1249-1278.

Farrell, Joseph, and Paul Klemperer. "Coordination and lock-in: Competition with switching costs and network effects." *Handbook of industrial organization* 3 (2007): 1967-2072.

Gandal, Neil & Michael Kende & Rafael Rob, 2000. "The Dynamics of Technological Adoption in Hardware/Software Systems: The Case of Compact Disc Players," *RAND Journal of Economics*, vol. 31(1), pages 43-61, Spring.

Gentzkow, Matthew & Jesse M. Shapiro, 2015. "Measuring the Sensitivity of Parameter Estimates to Sample Statistics," mimeo.

Greenstein, Shane, and Marc Rysman. 2007. "Coordination Costs and Standard Setting: Lessons from 56K Modems." In *Standards and Public Policy*, ed. S. Greenstein and V. Stango. *Journal of the European Economic Association*, 123-59. Cambridge University Press.

Jeitschko, Thomas D. and Tremblay, Mark J., Platform Competition with Endogenous Homing (April 10, 2015)

Lee, Robin. 2013. "Vertical Integration and Exclusivity in Platform and Two-Sided Markets." *American Economic Review*, 103(7): 2960-3000.

Mazzeo, M. (2002), "Product Choice and Oligopoly Market Structure", *Rand Journal of Economics* 33(2): 1-22.

Rochet, Jean-Charles and Jean Tirole (2003) "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 1(4), 990-1029.

Rysman, M. (2004), "Competition Between Networks: A Study of the Market for Yellow Pages", *Review of Economic Studies* 71: 483-512.

Rysman, M. (2009). The economics of two-sided markets. *The Journal of Economic Perspectives*, 125-143.

Rysman, Marc. 2007. "An Empirical Analysis of Payment Card Usage." *Journal of Industrial Economics*, 55(1): 1-36.

Seim, K. (2006), An empirical model of firm entry with endogenous product-type choices. *The RAND Journal of Economics*, 37: 619-640.

Sinkinson, Michael (2014) "Pricing and Entry Incentives with Exclusive Contracts: Evidence from Smartphones" Wharton School Working Paper, University of Pennsylvania.

Small, K. A. and H. S. Rosen. 1981. "Applied Welfare Economics with Discrete Choice Models," *Econometrica*, 49(1):105-130.

Yin, Pai-Ling, Jason P. Davis, and Yulia Muzyrya. 2014. "Entrepreneurial Innovation: Killer Apps in the iPhone Ecosystem." *American Economic Review*, 104(5): 255-59.

Zhu, Ting, Hongju Liu and Pradeep K. Chintagunta. 2011. "Wireless Carriers' Exclusive Handset

X GLOSSARY

Term	Definition
i	iOS (iPhone)
d	Android
b	both platforms
$*$	comScore variable
r^*	observed reach
r_{pa}^*	observed reach
r_{da}^*	observed reach
r_{ia}^*	observed reach
a	app
p	platform
S^*	the observed presence of the app on comScore
S_{ba}^*	observed multihoming
$mins_p^*$	minutes of use per user per month for app on platform
Game	the app is in the game category.
Mobile Only (MO)	The developer’s only line of business at firm founding involves mobile apps. (Some mobile-era firms, such as Rovio, have added other lines of business after entry, but we classify them as mobile-only by their status at entry.)
Online	The developer has an online business along with a mobile app (e.g., Facebook) and was, at time of entry, an online-only firm.
Offline	The developer had an offline business before having an online business or mobile app (e.g., Delta, Nike, other brick and mortar stores selling physical goods or service).
MG	Mobile Only Game
NG	Online Game
FG	Offline Game
MT	Mobile Only Other
NT	Online Other
FT	Offline Other

Term	Definition
X, X_a	regressors
(S, S^*)	publication status, e.g. (b, d)
π_p	profit $M_p \times U_p \times r_p - C_p$
M_p	per-customer margin earned by app a on platform p
C_p	fixed costs of writing and marketing
U_p	the number of users on platform p
$U_p \times r_{pa}$	the number of customers for the app on platform p
A_p	aggregate supply of apps to platform
$\bar{v}_p, \bar{v}$	intrinsic platform value(s)
$v_p = v(A_p)$	value of apps available on p
$\hat{r}_p \equiv C_p/(M_p \times U_p)$	entry threshold
$f_p(r)$	density of the distribution of r on p .
U_M	total number of users
$D_p(\bar{v}, v, U_M)$	users' platform demand
$\eta_{p'}^p$	elasticities of user demand for platforms
$\chi(U_d)$	function whose fixed point is an equilibrium
U_d^e	fixed point of $\chi(U_d)$, equilibrium
U_i^e	equilibrium
$\hat{r}_p^e$	equilibrium
$\tilde{r}$	potential reach –only an econometric, not a theoretical, concept
q_i, q_d and q_b	underlying distributions behind potential reach
R	signal of potential reach
ω	mixing parameter
$\text{beta}(\alpha_{iX_a}, \beta_{iX_a})$	parameters of potential reach distribution
param_{X_a}	param is a regression on parameters and X_a
$\bar{r}_p$	shrunk r if doesn't get through app store
$\delta, \delta_p X_a$	app store shrinkage parameter
r_p	realized demand for the app
κ_p	$= C_p/(M_p \times U_p)$ normalized fixed cost of supplying platform p .
κ_{pX_a}	
$f_Y(S_a, S_a^*, r_a^* X_a, \theta)$	(unconditional) likelihood for app a
$L_C(S, S^*, r^* X, \theta)$	conditional likelihood
v^{MH}	value of multihoming apps
$f(r)$	probability distribution of r

XI EMPIRICAL STABILITY CONDITION

XI.A

To see what underlying economics will lead to stability, we differentiate (2) to obtain $\partial v_p / \partial \hat{r}_p = -f_p(\hat{r}_p) \hat{r}_p$ and (1) to obtain $\partial \hat{r}_p / \partial U_p = -C_p / (M_p \times U_p^2) = -\hat{r}_p / U_p$. This lets us restate the slope of χ as

$$(6) \quad \chi' = \frac{\partial D_d()}{\partial v_d} f_p(\hat{r}_d) \hat{r}_d \frac{\hat{r}_d}{U_d} - \frac{\partial D_d()}{\partial v_i} f_p(\hat{r}_i) \hat{r}_i \frac{\hat{r}_i}{U_i}$$

Multiplying and dividing by v_p lets us convert to elasticities:

$$(7) \quad \chi' = \eta_d^d \frac{f_d(\hat{r}_d) \hat{r}_d}{v_d} \hat{r}_d - \eta_i^d \frac{f_i(\hat{r}_i) \hat{r}_i}{v_i} \hat{r}_i$$

Evaluating the stability condition at a symmetric equilibrium, we can further simplify (7) using the following properties: $U_i = U_d$, $\eta_d^d = -\eta_i^d$, and the elasticity of user demand between the two platforms η satisfies $\eta = \eta_d^d/2$. The elasticity of supply of user value from apps on a platform with respect to the threshold is $\eta_p^S = \partial v_p / \partial \hat{r}_p$.

XI.B

To calculate the empirical version of equation ?? based on our estimates we (a) update the formula to reflect the observable variety in applications (X) in our empirical model and (b) allow for the possibility that U_i and U_d are not equal.

The definition of an equilibrium does not change. The additional details appear in the definition of the mapping $\chi(U_d)$ whose fixed points are equilibria. Let W_X be the population number of potential developers of apps of type X . To avoid proliferation of symbols, we use the same notation as in the text, with subscript X meaning specific to developers/apps of type X and the notation $\{*_X\}$ to mean the set of all the objects $*$ indexed by X .

	Source	Defined Object	Definition
(8)	Identity	$U_i(U_d)$	$U_i = U_M - U_d$
	Eqn. (1)	$\hat{r}_{pX}(U_p)$	$\hat{r}_{px} = C_{pX} / (M_p \times U_p)$
	Eqn. (2)	$v_p(\{\hat{r}_{pX}\})$	$v_p = \sum_X W_X \int_{\hat{r}_{pX}}^1 r f_{pX}(r) dr$
	Eqn. (3)	$U_d(v_d, v_i)$	$U_d = D_d(\bar{v}, v_d, v_i, U_M)$

Recall the definition of local stability is $\chi'(U_d^e) < 1$. We have

$$\frac{\partial \chi(U_d)}{\partial U_d} = \frac{\partial D_d(\cdot)}{\partial v_d} \sum_X \left\{ W_X \frac{\partial v_{dX}}{\partial \hat{r}_{dX}} \frac{\partial \hat{r}_{dX}}{\partial U_d} \right\} + \frac{\partial D_d(\cdot)}{\partial v_i} \sum_X \left\{ W_X \frac{\partial v_{iX}}{\partial \hat{r}_{iX}} \frac{\partial \hat{r}_{iX}}{\partial U_i} \right\} \frac{\partial U_i}{\partial U_d}$$

Based on our estimates, it is easy to calculate the two sums. Under typical discrete-choice assumptions, $\partial D_d(\cdot) / \partial v_d = -\partial D_d(\cdot) / \partial v_i = \partial D_i(\cdot) / \partial v_i$. Define $\eta_{p'}^p$ as the elasticity of demand for platform p with respect to $v_{p'}$ by consumers. Let

$$A_p = \sum_X W_X \frac{\partial v_{pX}}{\partial \hat{r}_{pX}} \frac{\partial \hat{r}_{pX}}{\partial U_p} \frac{U_p}{v_p}$$

Then, for stability

$$\frac{\partial \chi(U_d)}{\partial U_d} \equiv \eta_d^d A_d - \eta_i^d A_i < 1$$

We can replace η_i^d using $\eta_i^d * D_d / v_i = -\eta_d^d * D_d / v_d$ which leads to our bound on the elasticity of demand.

Stability holds if

$$(9) \quad \frac{\partial \chi(U_d)}{\partial U_d} \equiv \eta_d^d \left\{ A_d + A_i \frac{v_i}{v_d} \right\} < 1$$

XI.C The Multidimensional Problem

To examine stability in cases with any number of platforms, we calculate the Jacobian of the function which maps U to U and whose fixed points are equilibria, $D(v(\hat{r}(U)))$. If the real part of all eigenvalues of the function are less than 1 in absolute value at an equilibrium, the equilibrium is stable.

What is the jacobian of $D(v(\hat{r}(U)))$? Definition: $J_D J_v J_{\hat{r}}$.

XI.D $J_{\hat{r}}$

In our specification, $\hat{r}_{Xp} = [\kappa_p(U_p) - (1 - \lambda_{Xp})\delta E[r]_{Xp}] / \lambda_{Xp}$.

Remember also that $\kappa_p \equiv C_p / (M_p \times U_p) = \kappa'_p / U_p$ so that $\partial \kappa / \partial U_p = -\frac{\kappa_p}{U_p}$

Then the dimensionality of $\hat{r}$ is that of Xp . An element of $J_{\hat{r}}$ is

$$J_{\hat{r}}[Xp] = -\frac{\kappa_p}{U_p} / \lambda_{Xp}$$

XI.E J_v

Second, with probability λ_{Xp} , the app sails through the app store; otherwise it gets a new draw but shrunk by δ_p . Thus

$$v_p(\hat{r}_p) = \sum_X w_X \left((1 - \lambda_{Xp})\delta E[r_{Xp}] + \lambda_{Xp} \int_{\hat{r}} f_{pX}(t, \theta) dt \right)$$

where the first term does not depend on $\hat{r}$ and the w_X are the *number* of developers in each X cell ("DEV W"). Thus

$$J_v[Xp] = -\sum_X w_X \lambda_{Xp} f_{pX}(\hat{r}_p, \theta) \hat{r}_p.$$

XI.F $J_v J_{\hat{r}}$

We can now get $J_v J_{\hat{r}}$ which is diagonal and $\dim(p)$ by $\dim(p)$.

Let $B_p(\hat{r}_p, \theta) = \sum_X w_X f_{pX}(\hat{r}_p, \theta) \hat{r}_{Xp}$.

$$J_v J_{\hat{r}} = \text{diag}[B_p(\hat{r}_p, \theta) \kappa_p / U_p]$$

Thus $J_v J_{\hat{r}}$ is diagonal. Also, it contains only parameters we have estimated.

XI.G J_D

J_D will, however, typically not be diagonal. We don't make estimates of D , so we need to make assumptions about it. Assume that D is a logit, and that the coefficient of v is β . That is, $U_p = U_M * \exp(d_p + \beta * v_p) / \sum_j \exp(d_j + \beta * v_j)$. The d_p are intercepts that let us predict any given candidate equilibrium; it is β that matters for stability.

Then $J_D[pp] = U_m \beta U_p / U_M (1 - U_p / U_M)$ and $J_D[pp'] = -U_M \beta U_p / U_M U_{p'} / U_M$. This assumption makes the demand slopes of unpopular platforms small, and thus tends to suggest that markets with unpopular platforms will be stable. We can see if this is reversed by looking at the rest of the model. Introducing P_p as the proportion of users who choose p , these can be written as $J_D[pp] = U_m \beta P_p (1 - P_p)$ and $J_D[pp'] = -U_M \beta P_p P_{p'}$.

XI.H η

We will bound β by asking for what values equilibrium will be stable. Since β itself is not an intuitive economic parameter, we will report the bound in terms of the elasticity of the user demand for iOS with respect to v_i , i.e. $\eta = \beta U_i / U_M v_i$. aka $\beta = \eta U_M / U_i / v_i$

XII DETAILS OF THREE CASES

We are interested in three special cases of this calculation.

XII.A *Historical Duopoly*

First special case, stability of the Android/iOS duopoly in the US. For this case, we assume that Android and iOS are the whole market, i.e., basically we assume that only folks who truly love WinMo or Blackberry choose those. So we have $U_M = U_i + U_d$. Another big plus is that we can use our estimates very directly to calculate $\hat{r}$ and so on.

We have

$$B_p(\hat{r}_p, \theta) = \sum_X w_X f_{pX}(\hat{r}_p, \theta) \hat{r}_{Xp}.$$

$$J_v J_{\hat{r}} = \text{diag}[B_p(\hat{r}_p, \theta) \kappa_p / U_p] \text{ and}$$

$$J_D = U_M \beta \begin{bmatrix} U_i / U_M (1 - U_i / U_M) & -U_i / U_M U_d / U_M \\ -U_i / U_M U_d / U_M & U_d / U_M (1 - U_d / U_M) \end{bmatrix}$$

$$J_D J_v J_{\hat{r}} = \beta \begin{bmatrix} (1 - U_i / U_M) B_i(\hat{r}_i, \theta) \kappa_i & U_d / U_M B_i(\hat{r}_i, \theta) \kappa_i \\ U_i / U_M B_d(\hat{r}_d, \theta) \kappa_d & (1 - U_d / U_M) B_d(\hat{r}_d, \theta) \kappa_d \end{bmatrix}.$$

XII.B Formula

In the unrestricted case, we have $J_D J_v J_{\hat{r}} = \beta \begin{bmatrix} (1 - U_i/U_M)B_i(\hat{r}_i, \theta)\kappa_i & U_d/U_M B_i(\hat{r}_i, \theta)\kappa_i \\ U_i/U_M B_d(\hat{r}_d, \theta)\kappa_d & (1 - U_d/U_M)B_d(\hat{r}_d, \theta)\kappa_d \end{bmatrix}$.

$$= \eta U_M/U_i/v_i \begin{bmatrix} (1 - U_i/U_M)B_i(\hat{r}_i, \theta)\kappa_i & U_d/U_M B_i(\hat{r}_i, \theta)\kappa_i \\ U_i/U_M B_d(\hat{r}_d, \theta)\kappa_d & (1 - U_d/U_M)B_d(\hat{r}_d, \theta)\kappa_d \end{bmatrix}.$$

XII.C Adding WinMo

To add WinMo, we of course need to move to a 3 by 3 Jacobian. We also need to specify the point at which we evaluate for all three platforms.

For d and i , the only thing that changes is U_M . For these two platforms, κ , $\hat{r}$, B , v_i all stay the same.

For w , our first counterfactual is $U_w = .19U_i$, and all the economic fundamentals from i are assigned to w .

Since $\kappa_p \equiv C_p/(M_p \times U_p) = \kappa'_p/U_p$, our assumption that $U_w = .5U_i$ gives us

$$\kappa_w = 2\kappa_i \text{ and}$$

$$\hat{r}_{Xw} = [\kappa_w - (1 - \lambda_{Xi})\delta E[r]_{Xi}] / \lambda_{Xi}.$$

f

$$B_w(\hat{r}_w, \theta) = \sum_X w_X f_{iX}(\hat{r}_w, \theta) \hat{r}_{Xw}$$